WHAT'S MATH GOOD FOR, WHAT CAN I DO WITH IT, AND WHY DO I EVEN CARE?

by
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A dissertation submitted to Johns Hopkins University in conformity with the requirements for the degree of Doctor of Philosophy

Baltimore, Maryland
August 2020

Abstract

Understanding mathematics proficiency requires looking beyond what students are able to do (and even how what they can do relates to the classroom context) to consider how the classroom context gives meaning to the ways that students are engaging with the content and one another. Students' ability to see value in mathematics, both in terms of importance and utility, and to position themselves as capable of achieving mathematical success is intimately tied to the degree to which they are able to develop in other aspects of mathematics proficiency: conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. The primary research objective of this dissertation is to illuminate the process by which dispositions towards mathematics develop, with particular consideration of how the classroom context can influence mathematics value and mathematics identity.

The first contribution of this dissertation is the development of a process model for mathematics disposition that considers observable elements of students' mathematical experiences alongside unmeasured, cognitive processes. The second contribution of this dissertation is the development of two latent scales, Deep Learning and Engagement Practices and Mathematics Comfort. The final contribution of this dissertation is the use of structural equation modeling, guided by the process model, to investigate the extent to which elements of the mathematics classroom context impact students' mathematics valuation (Study 1) and mathematics identity beliefs (Study 2) using survey data from a sample of 1,425 high school students collected during the 2017-2018 academic year.

The process model presented in this dissertation and the two investigations seek to demonstrate the mechanisms by which changes to students' mathematics motivation can be understood, and how certain practices and mathematical experiences provide a space for the negotiation of students' mathematics identities. Specifically, findings from the two studies suggest that the mathematical experiences provided to students in their mathematics courses, specifically those that are cognitively demanding, collaborative, and situated in real-world contexts, can increase students' valuation of and comfort with mathematics.

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Acknowledgments

My deep gratitude to my advisor, Doug Mac Iver, whose time and feedback have been instrumental to this work, and to the members of the faculty at Hopkins who taught, questioned, and pushed me.

I would also like to express my appreciation for my friends and family who provided encouragement and support throughout this process. In particular, I am eternally grateful to my mother for all that she has done to help me be successful, and for the sacrifices she has made so that I could have every opportunity available to me.

And finally, a thank you to IKEA of Schiltigheim, France.

DISCLAIMER

The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R305A150449 to the Johns Hopkins University. The opinions expressed are those of the author and do not represent views of the Institute or the U.S. Department of Education.

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CHAPTER I
The Multidimensionality of Mathematics Proficiency

In 2001, the National Research Council (NRC) released the report *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001) in which mathematical proficiency was specified as a multi-dimensional trait composed of five interdependent and interwoven strands: Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning and Productive Disposition. This joining of these distinct yet complementary strands presented in Figure 1.1 and detailed in Table 1.1, under one broad construct not only highlighted the relationships among the strands, but also underscored the individual contributions of each strand to mathematics proficiency. While four of the strands – Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning – pertain more to active employment of mathematics, Productive Disposition, defined as "the tendency to view mathematics as sensible, perceive it to be worthwhile and useful, and to view oneself as an effective learner and doer of mathematics" (NRC, 2001), speaks to an underlying motivational component, and that, in essence, students' mathematics beliefs serve as the lens through which they come to view mathematics learning and choose to engage in its application. The NRC explicitly recognized this dimension as its *own* strand of mathematical proficiency, not as an affective component of the other four, signalling a shift in the ways in which mathematical competency was to be defined and measured (Philipp & Siegfried, 2015).

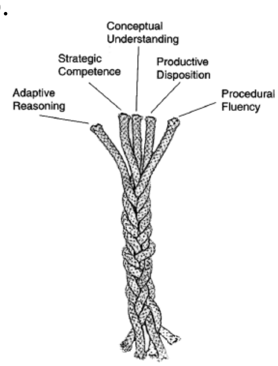

Figure 1.1. Strands of mathematical proficiency (NRC, 2001)

Table 1.1

The Strands of Mathematical Proficiency (National Research Council, 2001, p.115)

Strand	**Definition**
Conceptual Understanding	Comprehension of mathematical concepts, operations, and relations
Procedural Fluency	Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
Strategic Competence	Ability to formulate, represent, and solve mathematical problems
Adaptive Reasoning	Capacity for logical thought, reflection, explanation, and justification
Productive Disposition	Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Whether the NRC's presentation of mathematical proficiency was a call for a more holistic vision of what constitutes mathematical success, or an attempt to broaden the focus of mathematics teaching and learning to include consideration of the ways students experience and perceive mathematics, the productive disposition dimension of proficiency remains largely absent from most operational considerations of mathematics proficiency. A later publication by the National Council of Teachers of Mathematics (NCTM) of the educational principles that underlie each of the NRC's five strands of mathematical proficiency (NCTM, 2014) offers little discussion of ways to advance students' productive dispositions to mathematics (Grady, 2016). Few mathematics teachers are familiar with the term "productive disposition" (Siegfried, 2012), although they encounter students' productive (or counterproductive) dispositions every day, and that many of them actively seek to help their students develop more a productive disposition towards mathematics (Phillipp & Siegfried, 2015). The Common Core State Standards Initiative (CCSSI) introduced the Standards for Mathematical Practice (SMPs) that describe specific processes and proficiencies mathematics educators should seek to develop in students (CCSSI, 2010). These eight standards, often prominently displayed in mathematics classrooms, are a clear expansion and reworking of the NRC's first four strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. Only SMP1, to "make sense of problems and persevere in solving them" (CCSSI, 2010) partially addresses the fifth strand of productive disposition. This first SMP highlights perseverance, a behavior that could be attributed to a student who holds the belief that "steady effort in learning mathematics pays off", but excludes the other components of the NRC's definition, to "perceive [mathematics] as both useful and worthwhile" and "see oneself as an effective learner and doer or mathematics". An unintended consequence of the vast efforts dedicated to increasing students' mathematical competency along the first four strands is the resulting lack of consideration of and discussion around how students are coming to perceive mathematics. Grady (2016) calls for further examination of how instructional practices impact students' productive dispositions, and how these dispositions are related to the SMPs. Many studies that investigate teacher effects on student academic performance assess the effectiveness of teachers or the quality of teaching on the basis of the academic gains of their students (e.g., Aaronson et al., 2007, Sanders & Rivers, 1996). Such studies inadequately provide information on the instructional practices being employed (Stipek & Chiatovich, 2017) and the extent to which such practices support the development of productive dispositions towards mathematics. Yet students' ability to see value in mathematics, both in terms of importance and utility, and to position themselves as capable of achieving mathematical success is intimately tied to the degree to which they are able to develop in the other four strands. "The way we imagine, or 'mentally picture' what something *is* shapes our assumptions about what it is *for*, and this, in turn, informs our decisions in our efforts to achieve it" (Boulding, 1956, cited in Rutherford, 2015, p. 91). Rutherford then stresses the need to identify and critically reflect upon how educators picture and present the nature and purpose of mathematics. Scientific inquiry refers to a process of developing questions, designing experiments, collecting data, and making conclusions. It is a reasoning process used to solve problems, and it requires testing, validation, and evaluation (Lewis, 2006). Yet students often confound learning with the ability to supply correct answers (Barnett, 1992). Even among students majoring in fields related to mathematics, more than one half conceptualized the domain as number manipulation or a set of isolated techniques (Petocz et al., 2007).

Understanding Productive Disposition

Although well-designed paper-and-pencil tasks can measure conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning, assessing productive disposition with the same items may not be possible (Philipp & Siegfried, 2015). Students' written work does not necessarily provide clear indications of their effort (e.g., perseverance), or communicate whether or why they consider the mathematics involved to be interesting or useful. Researchers have only recently begun to develop instruments (e.g., Lewis et al., 2015) and approaches to investigate productive disposition in the broader context of mathematical proficiency (see Phillip & Siegfried, 2015 for an overview). Differences in how productive disposition is defined presents a challenge to this growing body of research. Based on the NRC's (2001) definition, a student with a productive disposition towards mathematics believes that, with effort, mathematics can be learned and applied effectively, that it is not arbitrary but understandable, and that is worthwhile and useful. Some researchers have broadened this definition to include other aspects or behaviors (Jacobson & Izsák, 2015; Lewis et al., 2015). In a study conducted by Gilbert (2014), productive disposition was operationalized using students' achievement goals and negative emotions, in addition to their task values and beliefs about their mathematics ability.

Identity and Disposition

While this dissertation maintains that mathematics value and ability beliefs are central elements of productive disposition, understandings of how to support productive disposition require consideration of how these elements relate to the broader learning context. In class, students are not only learning mathematics, but also what it means to *be* a learner and doer of mathematics (Boaler, 2002). There remains inadequate consideration in motivation research on how students are learning about mathematics or what is being taught (Turner et al., 2011).

Hand and Gresalfi (2015) consider identity as a joint construction between a person's *participation* in and across activities and the *understandings of oneself* that form in relation to those activities. In essence, this situative perspective considers identity to develop as one participates in an activity, and students alter their participation as they engage with resources differently. It is the joint accomplishment of an individual and their interactions with practices, cultural tools, norms, relationships, and the cultural and institutional contexts (Gee, 2000; Holland et al., 1998) that defines identity, and these interactions are both interpersonal (i.e., how one positions oneself or is positioned by others) and informational (i.e., how one utilizes the tools and practices of a discipline). Disposition, on the other hand, refers to the underlying mechanisms that give rise to events or practices. "Dispositions capture not only what one knows but how he or she knows it; and not only the skills one has acquired, but how those skills are leveraged" (Gresalfi, 2009, p.329). What an individual chooses to do in a particular activity is done in relation to both the resources that they bring from prior practices developed in other activities (Gutierrez & Rogoff, 2003) and what they have the opportunity to do (Greeno & MMAP, 1998). As such, dispositions are neither solely innate (i.e., individual traits or characteristics), nor solely the product of classroom practices. However, investigations of classroom activity systems and student participation (e.g., Cobb et al., 2009; Langer-Osuna, 2011), have demonstrated that observed forms of participation are frequently attributed to individual traits such as a student's intelligence or motivation. Deepening understandings around how to develop students' productive dispositions towards mathematics requires that research explicitly considers the context within which a learner is situated, and not continue to focus primarily on individual traits or characteristics.

Research Objective

The primary research objective of this dissertation is to illuminate the process by which dispositions towards mathematics develop, with particular consideration of how the mathematics classroom context can influence value beliefs and provide a space for the negotiation of students' mathematical identities. To accomplish this objective, this research

- presents a process model for mathematics disposition;
- collected primary data for key constructs of the process model over the course of one academic year;
- developed two latent scales;
- employed structural equation modeling, guided by the process model, to investigate the extent to which certain teacher practices and mathematical experiences support productive disposition (as defined by the National Research Council; NRC, 2001).

Conceptual Framework

This dissertation's conceptual framework (Figure 1.2) was developed so that observable elements of students' mathematical experiences could be considered alongside unmeasured, cognitive processes. By integrating theories of motivation and intelligence – expectancy value theory (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000) and achievement goal theory (Ames, 1992; Dweck & Leggett, 1988; Elliot & McGregor, 2001; Pintrich, 2000) – and incorporating the relationships and change mechanisms identified in self-regulation and mathematics motivation literature, this process model, termed the Mathematics Disposition Framework, provides a more comprehensive means of investigating the motivational patterns of students than if drawing from solely one theory. The framework illustrates the process by which (a) the mathematical learning environment is established, (b) mathematics value and identity beliefs advance, (c) engagement-related decisions are formed and enacted, and (d) performance outcomes are realized, with cognitive appraisals occurring throughout this process.

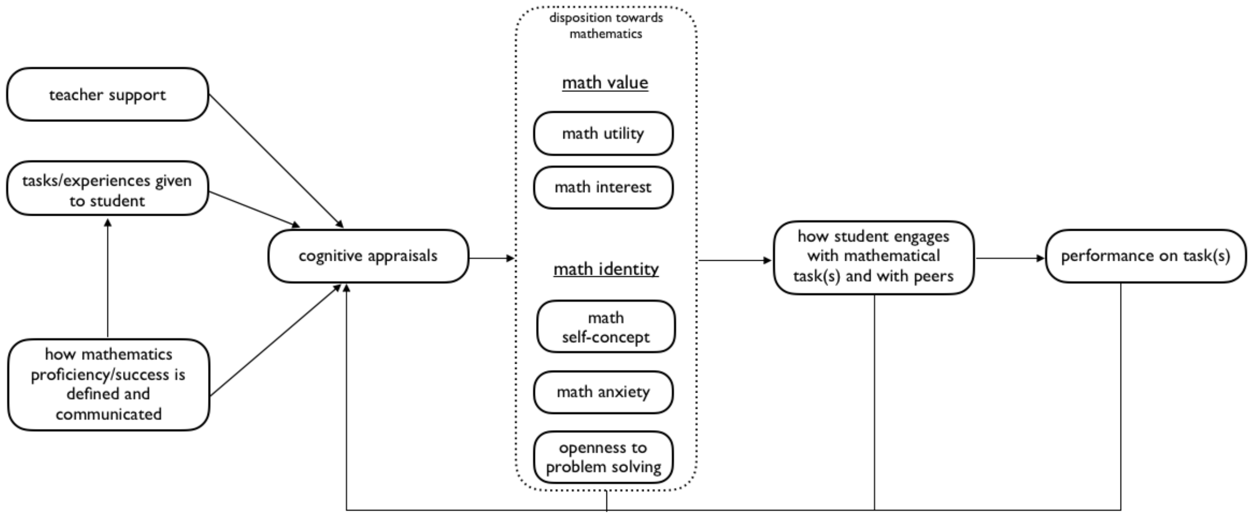

Figure 1.2. The Mathematics Disposition Framework. The framework does not include any established or hypothesized directional relationships among the mathematics value and identity constructs.

Classroom processes are dynamic and relational, involving the ongoing communication and interactions between students and teachers (Pianta & Hamre, 2009). Social and classroom factors (e.g., teacher instructional and grading practices), course expectations, and the ways in which students interact with course content and one other impact their strategic behaviors and engagement (Cleary & Chen, 2009; Eccles et al., 1993). Emotions are key predictors of achievement and self-regulation (Ahmed et al., 2013; Pekrun & Schutz, 2007), and self-motivational mathematics beliefs play a central role in promoting students' use of strategic and regulatory behaviors (Schunk et al., 2013). According to Boekaerts (2007), a match between tasks and personal goals produces positive emotions and cognitions that prompt students to utilize adaptive motivational and cognitive strategies, whereas a mismatch between the two results in negative emotions that lead students to display task-avoidance behaviors as a means of protecting self-image. Boekaerts posits that students' positive self-appraisal and affect lead to increased effort, which in turn results in increased performance. In essence, how mathematical success is defined and understood by students in conjunction with the tasks/experiences and support given to them impacts their self-regulation process. The goal-directed, interactive process of self-regulation involves cognitive appraisals of tasks and their congruence with an individual's learning goals. This in turn produces emotions, cognitions, and beliefs that dictate the ways in which students choose to engage with the learning tasks and ultimately perform on those tasks.

Achievement goal theory (Dweck & Leggett, 1988), which focuses on individuals' beliefs about the nature of their competence and their purposes for engaging in achievement-related behavior (Patrick et al., 2011), assumes that the primary influencers of students' motivation are their environment and their personal dispositions and beliefs (Ames, 1992). Achievement goals encapsulate a person's grounds for engaging in a task and yields a meaning system from which an individual interprets and reacts to events (Dweck, 1986; Dweck & Leggett, 1988; Smiley et al., 2016). Expectancy-value theory (Eccles & Wigfield, 2002; Wigfield & Eccles, 2000) suggests that achievement goals in a domain are dictated by an individual's perceived expectation for success in, and their perceived value or importance of, that domain, and that these goals orient individuals to tasks and influence their effort and cognitive processes during task engagement. Extensive research supports this theory, especially in mathematics (e.g., Crombie et al., 2005; Meece et al., 1990).

Dissertation Overview

There is a need to examine to what extent the ranges in student self-perceptions, task perceptions, and value constructs depend on the contextual features of mathematics classrooms. Examination of how the learning context gives meaning to the ways in which students are engaging with mathematics and with their peers offers the prospect of improving efforts to advance productive dispositions towards mathematics. In this chapter, I present the five strands of mathematics proficiency put forth by the NRC (2001) and discuss the importance of the fifth strand, productive disposition. I then present the Mathematics Disposition Framework that serves as the guiding process model for this research. Chapter 2 presents the data and measures used in two studies of productive disposition, including the development of two new scales: Deep Learning and Engagement Practices and Mathematics Comfort. Chapter 3 (Study 1) examines the impact of cognitively demanding teaching practices and three mathematical experiences (collaborative learning, project-based learning, and real-world connections to mathematics content) on students' mathematics value beliefs. Chapter 4 (Study 2) investigates the role of a collection of pedagogical practices and mathematical experiences (DLEPs) on students' mathematics identity beliefs. Chapter 5 presents the limitations of the two studies, discusses broader implications of findings, and suggests next steps for research on productive disposition.

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CHAPTER II
Data

The data used in this dissertation come from a quasi-experimental professional development (PD) study, *Engaging High School Students in Academic Work* (Mac Iver et al., 2020), which took place during the 2017-2018 school year. (*Note*: the primary research aim of the PD study was to measure the impacts of a 10-session professional development intervention on course-passing rates; this dissertation utilizes the student survey data to investigate the research objective outlined in Chapter 1.) Two Southern California public high schools, Ace High School and Victory High School (pseudonyms), sharing the same school district in adjacent communities, of similar size and with similar student demographics and course-passing rates, were chosen for the study based on the mutual expressed interest of school leadership. The district assigned Ace High School to be the intervention school (i.e., the school whose teachers received the professional development) and Victory High School to be the comparison school. School leaders at Victory High School had expressed a preference to defer the intervention until the following academic year after the school had completed its re-accreditation by the state, which was taking place during the 2017-2018 year.

During the first week of the academic year, students attending both high schools received an information letter detailing the PD study's purpose and components. The letter provided the parents/guardians of students the option to exclude their child from the study by returning the letter's opt-out form. Student withdrawal from the study was minimal and comparable at both sites. The Student Baseline Questionnaire (SBQ) was administered to students during the second week of school. Approximately two months into the school year, a second survey, the Student Mathematics Questionnaire (SMQ), was administered to students in their mathematics course. In mid-April 2018, students were administered a final survey, the Student End of Year

Questionnaire (SEOY), which was also completed in their mathematics course. Table 2.1 presents school characteristics along with the number of respondents for each student survey for each school site.

Table 2.1

School Characteristics for 2017-2018 Academic Year

	Ace High School	**Victory High School**
Graduation rate[a]	82%	81%
Suspension rate[a]	11%	10%
Enrollment[b]	1606	1679
Socioeconomically disadvantaged[a]	89%	89%
English Language Learners[a]	13%	12%
Students with disabilities[a]	18%	16%
Foster or homeless youth[a]	2.4%	2.6%
Latino/Hispanic	67%	65%
Black/African American	25%	22%
White	5%	9%
Asian	2%	3%
Mathematics sections offered[c] (non-SPED)	53	51
Integrated Math I	27 (51%)	20 (39%)
Integrated Math II	9 (17%)	11 (22%)
Integrated Math III	8 (15%)	9 (18%)
Advanced course offering	13%	18%
Mathematics teachers[d]	16	15
Teaching an advanced mathematics course	4	4
Teaching SPED mathematics	5	3
1st year teaching	7%	8%
2nd – 5th year teaching	33%	8%
6th – 10th year teaching	13%	23%
11+ years teaching	47%	62%
Questionnaires response rates		
Student Baseline Questionnaire (SBQ)	1197 (73%)	1248 (74%)
Student Mathematics Questionnaire (SMQ)	1131 (70%)	1217 (72%)
Student End of Year Questionnaire (SEOY)	1047 (65%)	1051 (63%)
SBQ, SMQ, and SEOY	702 (44%)	723 (43%)
No questionnaire data	147 (9%)	175 (10%)

Note. [a]Based upon enrollment data provided by the California Department of Education (2020). [b]Students enrolled for at least 160 days in 2017-2018. [c]Fall 2017. Categories are not mutually exclusive. Advanced course reflects any honors/accelerated or advanced placement (AP) mathematics course. SPED = special education. SPED mathematics denotes any self-contained mathematics course specifically for students with disabilities. [d]Courseload consists of at least one section of mathematics. Experience percentages based on 15 teachers for Ace High School and 13 teachers for Victory High School.

Measures

All mathematics belief scales were presented to students at the beginning and end of the school year on the SBQ and SEOY. Only one belief measure, mathematics interest, was also included on the SMQ. Mathematics classroom characteristics were measured for the fall and spring semesters on the SMQ and SEOY, with the exception of teacher support which was only included on the SMQ.

Established Scales

The three student questionnaires included established scales from the 2012 and 2015 Programme for International Student Assessment (PISA) Student Questionnaire (Organisation for Economic Co-operation and Development [OECD], 2013, 2017), the Attitudes toward Mathematics Instrument (ATMI; Tapia, 1996), and the 2012 National Survey of Science and Mathematics Education (NSSME; Banilower et al., 2013). Table 2.2 displays each adopted construct, its origin, and the internal consistency reliability values by survey, based on all available cases in this dissertation's data sample.

Table 2.2

Constructs and Internal Consistency Values by Questionnaire

	Mathematics Interest	Mathematics Utility	Openness to Problem Solving	Mathematics Self- Concept	Mathematics Anxiety	Teacher Support	Reform-Oriented Practices	Cognitive Activation in Math Class
Scale origin	PISA 2012	ATMI	PISA 2015	PISA 2012	PISA 2012	PISA 2012	NSSME 2012 - adapted	PISA 2012
Pertaining survey(s) and corresponding scale internal consistency value	SBQ .89 SMQ .87 SEOY .89	SBQ .93 SEOY .92	SBQ .88 SEOY .81	SBQ .86 SEOY .85	SBQ .84 SEOY .85	SMQ .90	SMQ .74 SEOY .83	SMQ .86 SEOY .90

Note. PISA = Programme for International Student Assessment; ATMI = the Attitudes toward Mathematics Instrument; NSSME = National Survey of Science and Mathematics Education.

Beliefs. Table 2.3 presents the individual questionnaire items that compose each belief scale, along with the response options provided to students and the means and standard deviations for each timepoint the scale was used. Students were asked to read and respond to a series of statements (e.g. "*I am interested in the things I learn in mathematics*"), designed to capture the belief constructs of mathematics interest, mathematics utility, mathematics self-concept, mathematics anxiety, and openness to problem-solving. Students were asked to respond using a Likert scale from either '1' to '4,' or '1' to '5' depending on the construct. So that higher scores on each item corresponded with higher levels of the construct, one self-concept item that was negatively worded ("*I am just not good at mathematics*") was reverse coded.

Scale origins. The Mathematics Interest, the Mathematics Self-Concept, and the Mathematics Anxiety scales, all taken from the PISA 2012 Student Questionnaire (OECD, 2013), are comprised of four, five, and five items, respectively. (*Note*: The PISA 2012 Student Questionnaire is henceforth referred to as PISA 2012.) The Mathematics Utility scale, taken from the ATMI (Tapia, 1996), is comprised of ten items, however the last item ("*I want to develop my mathematical skills*") was accidentally left off of the SBQ. The Openness to Problem Solving scale, taken from the PISA 2015 Student Questionnaire (OECD, 2017), is comprised of five items.

Table 2.3

Items Comprising the Mathematics Interest, Mathematics Utility, Mathematics Self-Concept, Mathematics Anxiety, and Openness to Problem Solving Scales

SBQ M (SD)	SMQ M (SD)	SEOY M (SD)	
Mathematics Interest			**Thinking about your views on mathematics, to what extent do you agree with the following statements?** 1 (strongly disagree), 2 (disagree), 3 (agree), 4 (strongly agree)
2.0 (.77)	2.7 (.91)	2.0 (.78)	*I enjoy reading about mathematics.*
2.3 (.87)	2.7 (.88)	2.3 (.85)	*I look forward to my mathematics lessons.*
2.1 (.87)	2.3 (.95)	2.1 (.89)	*I do mathematics because I enjoy it.*
2.4 (.91)	2.0 (.86)	2.4 (.90)	*I am interested in the things I learn in mathematics.*
Mathematics Utility			**Thinking about your views on mathematics, to what extent do you agree with the following statements?**[a] 1 (strongly disagree), 2 (disagree), 3 (neutral), 4 (agree), 5 (strongly agree)
3.5 (1.1)		2.9 (.84)	*Mathematics is a very worthwhile and necessary subject.*
3.6 (1.0)		3.0 (.75)	*Mathematics helps develop the mind and teaches a person to think.*
3.5 (1.1)		2.9 (.82)	*Mathematics is important in everyday life.*
3.3 (1.1)		2.8 (.83)	*I believe studying math helps me with problem solving in other areas.*
3.4 (1.1)		2.8 (.81)	*High school math courses would be very helpful no matter what I decide to study.*
3.3 (1.2)		2.7 (.87)	*I can think of many ways that I use math outside of school.*
3.2 (1.1)		2.7 (.87)	*I think studying advanced mathematics is useful.*
3.4 (1.1)		2.8 (.84)	*Mathematics is one of the most important subjects for people to study.*
3.6 (1.1)		2.9 (.82)	*A strong mathematics background could help me in my professional life.*
		3.0 (.82)	*I want to develop my mathematical skills.*[b]
Mathematics Self-Concept			**Thinking about studying mathematics, to what extent do you agree with the following statements?** 1 (strongly disagree), 2 (disagree), 3 (agree), 4 (strongly agree)
2.5 (.96)		2.5 (.96)	*I am just not good at mathematics.*[c]
2.7 (.82)		2.6 (.87)	*I get good grades in mathematics.*
2.4 (.89)		2.5 (.87)	*I learn mathematics quickly.*
2.3 (1.01)		2.3 (1.00)	*I have always believed that mathematics is one of my best subjects.*
2.3 (.88)		2.3 (.89)	*In my mathematics class, I understand even the most difficult work.*
Mathematics Anxiety			**Thinking about studying mathematics, to what extent do you agree with the following statements?** 1 (strongly disagree), 2 (disagree), 3 (agree), 4 (strongly agree)
2.9 (.88)		2.8 (.90)	*I often worry that it will be difficult for me in mathematics classes.*
2.5 (.88)		2.5 (.90)	*I get very tense when I have to do mathematics homework.*
2.5 (.89)		2.4 (.89)	*I get very nervous doing mathematics problems.*
2.3 (.88)		2.3 (.92)	*I feel helpless when doing a mathematics problem.*
2.8 (1.00)		2.8 (1.01)	*I worry that I will get poor grades in mathematics.*
Openness to Problem Solving			**How well does each of the following statements describe you?** 1 (not at all like me), 2 (not much like me), 3 (somewhat like me), 4 (mostly like me), 5 (very much like me)
3.3 (.93)		3.4 (.95)	*I can handle a lot of information.*
3.3 (1.03)		3.4 (1.01)	*I am quick to understand things.*
3.7 (1.06)		3.7 (1.01)	*I seek explanations for things.*
3.4 (1.04)		3.4 (1.02)	*I can easily link facts together.*
2.9 (1.19)		3.0 (1.16)	*I like to solve complex problems.*

Note. [a]The "neutral" response option was accidentally omitted from the SEOY survey, so the SEOY Mathematics Utility response options were as follows: 1 (strongly disagree), 2 (disagree), 3 (agree), 4 (strongly agree). [b]This item was inadvertently omitted from the SBQ. [c]Response option scores were inverted.

Mathematics classroom characteristics measures. Table 2.4 displays the survey items pertaining to all mathematics classroom characteristics measures. Students were asked to read and respond to a series of statements regarding pedagogical practices employed by their mathematics teacher (e.g., "*the teacher has us compare and contrast different methods for solving a problem*"), qualities of their mathematics teacher (e.g., "*the teacher shows an interest in every student's learning*"), or of specific mathematical experiences that were taking place in that mathematics course (e.g., "*students work on projects that require several days to complete*"). Students were asked to indicate how often each occurred in their mathematics course. Using a Likert scale ranging from either '1' to '4' or '1' to '5', each item was presented such that a score of 1 reflected the lowest frequency of occurrence. For example, the item "*the teacher asks us to decide on our own procedures for solving complex problems*" presented students with the following response options: '1' (never or almost never), '2' (some lessons), '3' (most lessons), and '4' (almost every lesson). The Teacher Support and the Cognitive Activation in Math Class scales were taken from PISA 2012, and are comprised of four and nine items, respectively. The Reform-Oriented Teaching Practices scale from the 2012 NSSME was adapted for students for use in this study (the NSSME scale was intended for teacher respondents). Three PISA 2012 items not part of an established scale were also included: "*the teacher makes connections between the math and real-world situations or applications*," "*the teacher has us work in small groups to come up with joint solutions to a problem or task*," and "*students work on projects that require several days to complete*" (adapted from the original PISA 2012 wording "*the teacher assigns projects that require at least one week to complete*").

Table 2.4

Mathematics Classroom Characteristics Items

	SMQ M (SD)	SEOY M (SD)	
Reform-Oriented Practices			**How often do these things happen in this mathematics class?** 1 (never), 2 (rarely – a few times a year), 3 (sometimes – once or twice a month), 4 (often – once or twice a week), 5 (all or almost all lessons)
RF1	4.1 (1.12)	4.0 (1.18)	*The teacher has us explain and justify our method for solving a problem.*
RF2	4.0 (1.10)	4.0 (1.08)	*The teacher has us consider multiple representations in solving a problem (for example: numbers, tables, graphs, pictures).*
RF3	3.1 (1.41)	3.3 (1.36)	*The teacher asks us to present our solution strategies to the rest of the class.*
RF4	3.4 (1.28)	3.6 (1.25)	*The teacher has us compare and contrast different methods for solving a problem.*
Cognitive Activation in Math Class			**About the activities and tasks in your mathematics lessons:** 1 (never or almost never), 2 (some lessons), 3 (most lessons), 4 (almost every lesson)
CA1	2.8 (.88)	2.8 (.90)	*The teacher asks questions that make us reflect on the problem.*
CA2	2.9 (.86)	3.0 (.84)	*The teacher gives problems that require us to think for an extended time.*
CA3	2.4 (.96)	2.6 (.94)	*The teacher asks us to decide on our own procedures for solving complex problems.*
CA4	2.7 (.93)	2.9 (.89)	*The teacher gives problems that can be solved in several different ways.*
CA5	3.1 (.94)	3.1 (.91)	*The teacher helps us learn from the mistakes we have made.*
CA6	2.9 (.94)	2.9 (.91)	*The teacher presents problems in different contexts so that students know whether they have understood the concepts.*
CA7	3.0 (.96)	3.0 (.93)	*The teacher asks us to explain how we have solved a problem.*
CA8	2.9 (.89)	3.0 (.89)	*The teacher presents problems that require students to apply what they have learned to new contexts.*
CA9	2.6 (.92)	2.7 (.92)	*The teacher presents problems for which there is no immediately obvious method of solution.*
Teacher Support			**How often do these things happen in this mathematics class?** 1 (never), 2 (rarely – a few times a year), 3 (sometimes – once or twice a month), 4 (often – once or twice a week), 5 (all or almost all lessons)
TS1	3.8 (1.3)	-	*The teacher gives students an opportunity to express opinions.*
TS2	4.2 (1.1)	-	*The teacher gives extra help when students need it.*
TS3	4.0 (1.2)	-	*The teacher continues teaching until the students understand.*
TS4	4.0 (1.2)	-	*The teacher shows an interest in every student's learning.*
TS5	4.3 (1.1)	-	*The teacher helps students with their learning.*
Other (non-scale)			**How often do these things happen in this mathematics class?** 1 (never), 2 (rarely – a few times a year), 3 (sometimes – once or twice a month), 4 (often – once or twice a week), 5 (all or almost all lessons)
RW	3.5 (1.34)	3.6 (1.28)	*The teacher makes connections between the math and real-world situations or applications.*
PRJ	2.1 (1.24)	2.6 (1.35)	*Students work on projects that require several days to complete.*
GRP	3.2 (1.38)	3.4 (1.34)	*The teacher has us work in small groups to come up with joint solutions to a problem or task.*

Note. The codes in the first column (e.g., RF1) assigned to each item are used again in Tables 2.5 and 2.6 and in Figure 2.1 (if applicable).

Suitability of Higher-Order Scales

Deep Learning and Engagement Practices. The NCTM (2014) put forth eight research-informed Mathematical Teaching Practices (MTPs) as essential to mathematics lessons in supporting the learning and engagement of all students at the highest possible level. Table 2.5 presents the mathematics classroom characteristics items that are considered to correspond with each MTP based upon the NCTM's descriptions of each practice (NCTM, 2014, 2018).

Table 2.5

NCTM Mathematical Teaching Practice and Associated Mathematics Classroom Characteristics Items

	NCTM Mathematical Teaching Practice	Questionnaire Items
1	Establish mathematics goals to focus learning.	GRP CA6
2	Implement tasks that promote reasoning and problem solving.	RF1 RF4 PRJ CA1 CA2 CA3 CA4 CA8 CA9 GRP
3	Use and connect mathematical representations.	RF2 RF4 RW CA6 CA8
4	Facilitate meaningful mathematical discourse.	RF1 RF3 RF4 GRP CA1 CA7
5	Pose purposeful questions.	RF1 RF3 RF4 CA1 CA3 CA7
6	Build procedural fluency from conceptual understanding.	RF2 RF4 CA3 CA5 CA6
7	Support productive struggle in learning mathematics.	PRJ CA1 CA2 CA5 CA9 GRP
8	Elicit and use evidence of student thinking.	RA1 RA3 CA1 CA2 CA3 CA7 CA8 CA9 GRP

Note. Teacher Support scale items are not included in this table, as they were not hypothesized to be indicators of a higher-order factor.

An Exploratory Factor Analysis (EFA) was conducted with the sixteen mathematics classroom characteristics items hypothesized to load onto a higher-order "deep mathematics learning and engagement" factor using SMQ data. Of the 2,348 students who completed the SMQ, a total of 108 respondents were counted as having at least one missing value and were removed. To assess suitability of data for factor analysis, Bartlett's Test of Sphericity was conducted and was significant ($p < .001$), evidencing the patterned relationships within the data. The data sample of 2,240 complete cases is sufficient for EFA (cases-to-variable ratio is greater than 5:1; Williams et al., 2010), confirmed with a Kaiser-Meyer-Olkin (KMO) statistic of .94 (.50 is considered suitable for factor analysis; Hair et al., 1995; Tabachnick & Fidell, 2012). Factors were extracted using principal components analysis, with the number of factors determined by eigenvalues and a scree test. An orthogonal promax rotation was applied to the EFA models as they produce correlated factors (Costello & Osborne, 2005). The purpose of rotating factors is to acquire an optimal simple structure in which each indicator loads onto as few factors as possible, yet also maximizes the number of strong loadings (Yong & Pearce, 2013). Items with factor loadings of .40 or greater were considered to load on that factor. Results of the principal components analysis suggested a one- or two-factor solution. A scree test indicated that a one-factor solution would be more appropriate (Williams et al., 2010). When the data were fit to the one-factor model, fifteen of the sixteen items loaded strongly onto the factor. The poorly loading item "*students work on projects that require several days to complete*" [PRJ] was dropped. One possible explanation as to why this item loaded less strongly onto the factor is because this item lumps together many types of projects that require several days to complete (e.g., projects that involve reasoning, problem-solving, and productive struggle as well as projects that involve a series of low-level tasks that don't necessarily promote deep learning and engagement). The one-factor solution accounted for 36% of the variance. The factor was named Deep Learning and Engagement Practices (DLEPs).

To indicate whether the hypothesized one-factor model was a good fit to the observed data, Confirmatory Factor Analyses (CFAs) were conducted using the *lavaan* package version 0.5-23 (Rosseel, 2012) in R software version 3.4.3 (R Core Team, 2017) using the SEOY survey responses ($n = 2{,}064$). Before conducting the CFAs, the data was examined for potential concerns (normality, missing cases). Full Information Maximum Likelihood (FIML) estimation was used to handle the missing data. Latent factors were standardized to allow for free estimation of all factor loadings. A CFA was first estimated without any error covariances between the DLEPs items. The following fit indices were used to assess model fit: Chi-squared test statistic (x^2) to degrees of freedom (df) ratio (< 5, Bollen, 1989), comparative fit index (CFI) ($\geq$ .90, Hu & Bentler, 1999), Tucker-Lewis fit index (TLI) ($\geq$ .90, Hu & Bentler, 1999), root-mean-square error of approximation (RMSEA) (< .05, Browne & Cudeck, 1992), and standardized root mean square residual (SRMR) (< .08, Hu & Bentler, 1999). Fit indices of the model were as follows: $x^2/df = 19.8$, CFI = .887, TLI = .868, RMSEA = .094, and SRMR = .050, indicating that the hypothesized model did not adequately fit the observed data. Modification indices were then calculated in order to assess the improvement to the model should error covariances between variables be added. Error covariances were added to the CFA model only if there was a strong theoretical justification (i.e., an additional relationship between two variables beyond a shared latent factor was expected, for example, teachers who ask students "*to present our solution strategies to the rest of the class*" [RF3] can then follow up by asking students "*to compare and contrast methods for solving a problem*" [RF4]). Model fit was greatly improved for the augmented CFA model: $x^2/df = 10.3$, CFI = .952, TLI = .935, RMSEA = .066, and SRMR = .036. Figure 2.1 presents the CFA model with error covariances. The factor loadings for the EFA and CFA models are provided in Table 2.6, along with the Cronbach's alpha for each factor. As expected, the indicators all showed significant positive factor loadings, with standardized coefficients that ranged from .564 to .777. The DLEPs measure is used in Study 2.

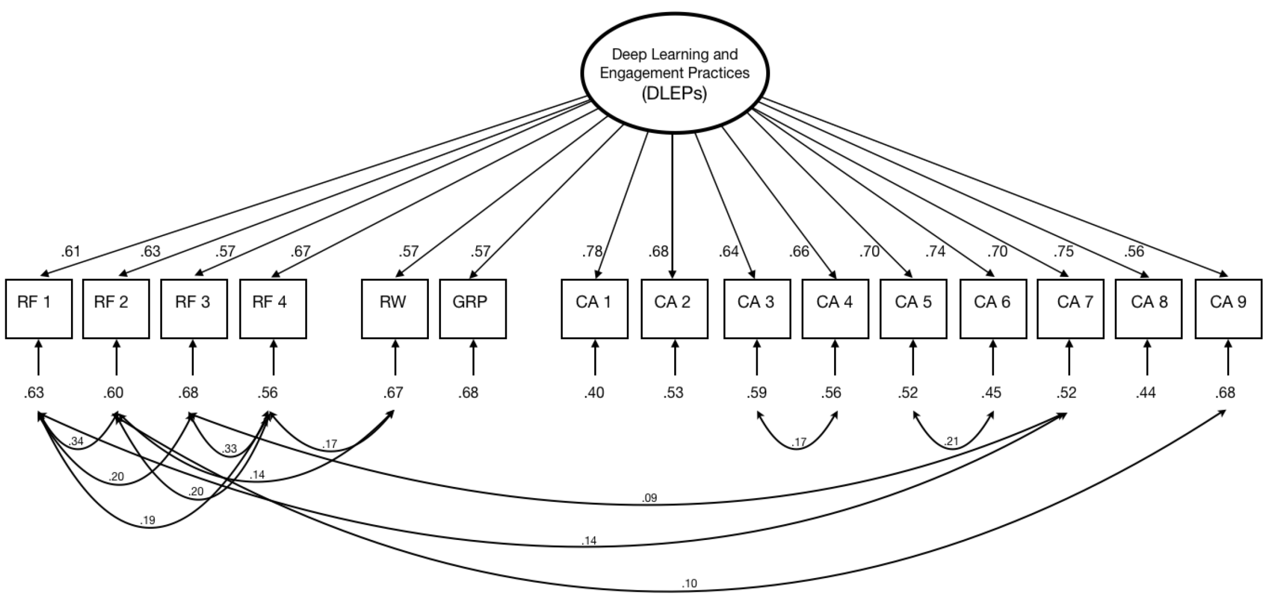

Figure 2.1. The Deep Learning and Engagement Practices scale.

Table 2.6

Factor Loadings of the Exploratory and Confirmatory Factor Analyses of the Deep Learning and Engagement Practices Items

Deep Learning and Engagement Practices

Item	**[a]EFA**	**[b]CFA 1**	**[c]CFA 2**
The teacher…			
…asks questions that make us reflect on the problem. [CA1]	.715	.768	.777
…presents problems that require students to apply what they have learned to new contexts. [CA8]	.649	.730	.751
…presents problems in different contexts so that students know whether they have understood the concepts. [CA6]	.698	.738	.744
…helps us learn from the mistakes we have made. [CA5]	.640	.696	.697
…asks us to explain how we solved a problem. [CA7]	.655	.697	.696
…gives problems that require us to think for an extended time. [CA2]	.571	.670	.684
…has us compare and contrast different methods for solving a problem. [RF4]	.620	.705	.665
…gives problems that can be solved in several different ways. [CA4]	.594	.658	.661
…asks us to decide on our own procedures for solving complex problems. [CA3]	.597	.638	.644
…has us consider multiple representations in solving a problem (for example: numbers, tables, graphs, pictures). [RF2]	.573	.673	.631
…has us explain and justify our method for solving a problem. [RF1]	.573	.657	.607
…makes connections between the math and real-world situations or application. [RW]	.511	.603	.574
…has us work in small groups to come up with joint solutions to a problem or task. [GRP]	.535	.578	.569
…asks us to present our solution strategies to the rest of the class. [RF3]	.517	.613	.567
…presents problems for which there is no immediately obvious method of solution. [CA9]	.508	.554	.564
n	2,240	2,064	
Cronbach's ∝	.89	.92	
Mean	3.1	3.2	
Standard Deviation	.67	.73	

Note. Standardized factor loadings are reported. EFA = Exploratory Factor Analysis; CFA = Confirmatory Factor Analysis. [a]EFA of fall DLEPs items. [b]CFA of spring DLEPs with no error covariances between items; [c]CFA of spring DLEPs with inclusion of error covariances between selected items.

Mathematics comfort. The PISA 2012 *mathematics anxiety* and *mathematics self-concept* scales were measured on the SBQ and the SEOY. Consistent with theory (Bandura, 1997; Pekrun, 2006) and empirical research (Ahmed et al., 2012; Frenzel, Pekrun, & Goetz, 2007; Goetz et al., 2006; Hembree, 1990; Meece et al., 1990; Parajes & Miller, 1994) that has documented the strong, inverse relationship between mathematics anxiety and mathematics self-concept, moderate to strong correlations in the data between the baseline self-concept and anxiety items were observed (rs ranged from –.27 to –.61, $p < .001$). To assess the suitability of a higher-order, latent "comfort with mathematics" variable, EFAs of the ten anxiety and self-concept items with the SBQ data ($n = 2196$), followed by CFAs of the same ten items with the SEOY data ($n = 2141$) were conducted. All items were coded such that a '1' reflected the smallest amount of mathematics comfort (low self-concept/high anxiety) and a ‘5’ reflected the highest (high self-concept/low anxiety).

As with the EFA of the DLEPs factor indicators, an orthogonal promax rotation was applied to the EFA of the mathematics anxiety and mathematics self-concept items due to the correlated nature of the factors (Costello & Osborne, 2005). Factors were extracted using principal components analysis. Eigenvalues, a scree test, and model fit comparisons were used to determine the number of factors to retain.

The principal components analysis indicated that there were two eigenvalues greater than 1 (4.41 and 1.11), and the scree test indicated that a one- or two-factor solution could be appropriate (Williams et al., 2010). As a result, an exploratory factor analysis of both a two-factor and one-factor model was tested. Items with factor loadings of .40 or greater were considered to load onto that factor. The two-factor solution accounted for 55.7% of the observed variance; the one-factor solution accounted for 46.9% of the observed variance. When the data were fit to a two-factor model, the items did not load onto the original constructs of mathematics anxiety and mathematics self-concept; all anxiety items as well as one negatively worded self-concept item “*I am just not good at mathematics*” [SC1] loaded together onto one factor, and the four positively worded self-concept items loaded onto the second. When the data were fit to a one-factor model, all ten items loaded strongly onto one factor.

To test the factor structure of the ten items, a CFA for both the one-factor and two-factor solutions was conducted and model fit comparisons were made. Model comparisons revealed that improvements to model fit when a two-factor solution was specified were negligible over the one-factor model ($\Delta x^2/df = -1.44$; ΔCFI $= -.001$; ΔTLI $= .007$; ΔRMSEA $= -.005$; ΔSRMR $= .001$). The one-factor solution was retained, and the latent variable, presented in Figure 2.2, was named Mathematics Comfort. The mathematics comfort measure is used in Study 2.

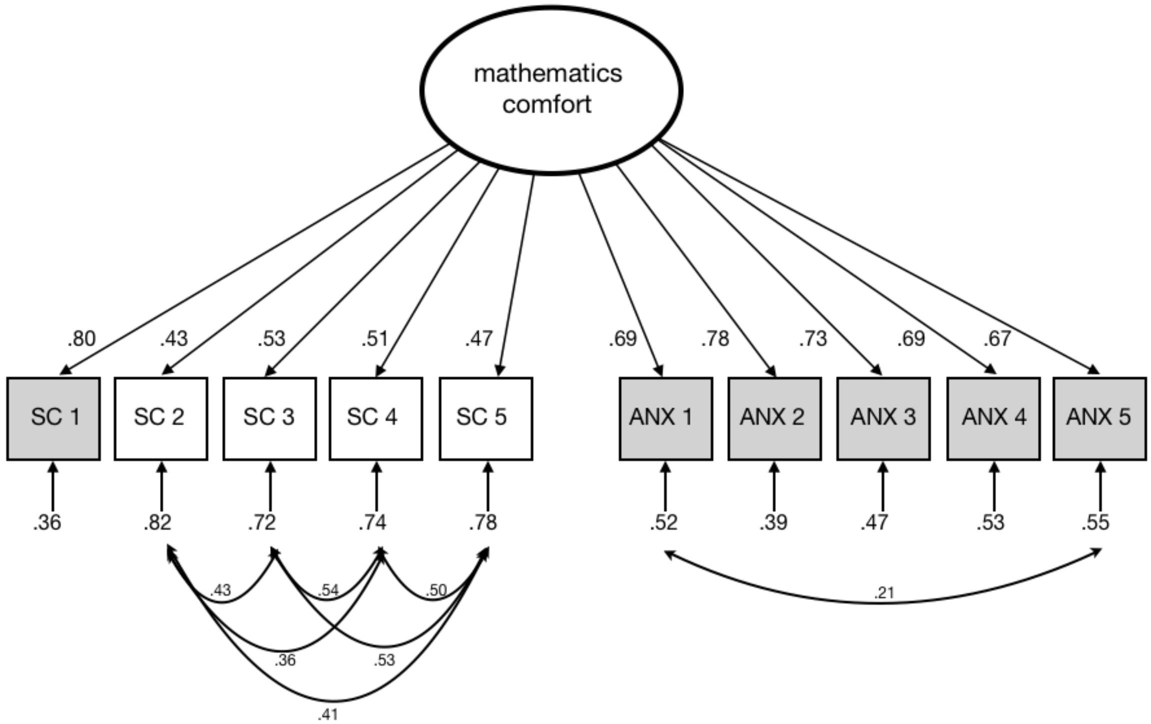

Figure 2.2. Single-factor model of mathematics comfort scale. Shaded boxes represent reverse-scored items. n = 2,141. x^2/df = 12.7; CFI = .968; TLI = .948; RMSEA = .074; SRMR = .033.

Other Student Characteristics

Mathematics course grade. District-reported mathematics course grades from the fall 2017 semester were utilized. Letter grades were converted to a four-point scale. Conversions to numeric grade-point values followed the method employed by Stanford University in which an A+ is valued at 4.3, A at 4.0, A– at 3.7, etc. (continuing the pattern down through the letter grade of D-, valued at 0.7). An F grade, as well as any less traditional letter assignments indicating course failure (e.g., F+, F-, NP), are valued at 0.0.

Gender and school. District-reported gender was utilized. Male and female were the only categories for gender. In all analyses, the reference category for gender is male. School was coded such that ‘1’ = Ace High School, and ‘0’ = Victory High School.

Advanced math. The "advanced math" designation is an indicator of whether or not a student was enrolled in an Advanced Placement (AP) or other form of accelerated/honors mathematics course (1 = advanced course, 0 = not advanced course).

Off track in math. The "off track in math" designation is an indicator of a student's mathematics trajectory (1 = off track, 0 = not off track). This variable was created using each student's mathematics course and grade level, and reflects whether or not a student is taking a particular mathematics course later than the expected mathematics course progression for that school. The variable of mathematics course was originally measured on the SMQ and SEOY categorically; one category was assigned to each unique course offering. A numeric designation aligned with the grade level was assigned to each course based on the grade in which students would be expected to take that course (e.g., the course Integrated Math II is intended for the tenth grade year, and was therefore assigned a 10; AP Calculus is typically taken in the twelfth grade, and was therefore assigned a 12). The difference between this numeric course attribution and each student's grade was then computed. As an example, the difference calculation for an eleventh grade student in Integrated Math II, a tenth grade course, would be –1, reflecting that this student is one year behind where they should be. Only students that received a negative difference calculation value were coded as '1' for the off track in math measure.

Data Analytic Strategy for Structural Equation Modeling

As structural equation modeling (SEM) allows for the simultaneous testing of multiple hypothesized associations among latent and observed variables, this dissertation employed SEM to investigate the relationships between students' classroom experiences and aspects of their productive dispositions towards mathematics (operationalized in this work by their valuation of mathematics and their comfort with mathematics). All models were estimated in R software version 3.4.3 (R Core Team, 2017). Standard assumptions of homoscedasticity, normality, and linearity were assessed through residual plots and histograms. The same fit indices used for the CFAs of the higher-order DLEPs and mathematics comfort scales were used to assess overall model fit for the SEM models: x^2/df (< 5), CFI (> .90), TLI (> .90), RMSEA (< .05), and SRMR (< .08). Single-level models were employed as the small number of teachers in conjunction with the large number of estimated parameters would lead to difficulties with model convergence and insufficient statistical power had multilevel structural equation modeling been employed.

Missing Data

Of the 3,285 students enrolled at Ace High School or Victory High School in 2017-2018, forty-three percent ($n = 1425$) completed all three student questionnaires. Table 2.7 compares this subsample of students with the full sample. The subsample was found to contain a significantly greater proportion of ninth grade students, female students, Latino/Hispanic students, and students in an advanced mathematics course compared with the full sample of students attending the two schools. There were significantly smaller proportions of twelfth grade students, students with disabilities, Black/African American students, and students behind in mathematics trajectory (off track designation) in the subsample. For the students who completed all three surveys, the average days present over the course of the 2017-2018 school year was 2.21 days greater, and the mean fall mathematics course grade .27 points higher (which roughly translates to a quarter of a letter grade). No significant differences were observed for the proportion of students attending each school, tenth or eleventh graders, White students, or Asian students. Among the mathematics beliefs and classroom experiences measures, only mathematics utility at baseline was observed to be significantly different between the subsample and the full sample (.06 points higher for the subsample).

Table 2.7

Full Sample to Subsample Comparison

		Full Sample		Subsample		
Variable	Range	*M* (*SD*)	*N*	*M* (*SD*)	*n*	*t*
School (1 = Ace)	0–1	0.49 (−)	3285	0.49 (−)	1425	0.20
Gender (1 = female)	0–1	0.47 (−)	3285	0.52 (−)	1422	3.54***
Grade 9	0–1	0.25 (−)	3285	0.29 (−)	1425	3.09**
Grade 10	0–1	0.26 (−)	3285	0.29 (−)	1425	2.25*
Grade 11	0–1	0.24 (−)	3285	0.25 (−)	1425	0.92
Grade 12	0–1	0.26 (−)	3285	0.18 (−)	1425	-8.39***
Attendance (days present)	1–180	171.74 (10.20)	3285	173.95 (6.81)	1421	12.34***
Special Education	0–1	0.17 (−)	3285	0.11 (−)	1425	-6.90***
Latino/Hispanic	0–1	0.66 (−)	3285	0.71 (−)	1425	3.87***
Black/African American	0–1	0.23 (−)	3285	0.17 (−)	1425	-5.65***
White	0–1	0.07 (−)	3285	0.07 (−)	1425	-0.29
Asian	0–1	0.02 (−)	3285	0.03 (−)	1425	1.82
Off Track in Math	0–1	0.32 (−)	2332	0.23 (−)	1425	-7.84***
Advanced Math	0–1	0.17 (−)	2332	0.24 (−)	1425	6.30***
F Mathematics Course Grade	0–4.3	1.99 (1.35)	3006	2.26 (1.31)	1418	7.71***
BL Mathematics Comfort	1–4	2.44 (0.65)	2307	2.46 (0.66)	1344	0.98
S Mathematics Comfort	1–4	2.44 (0.65)	1954	2.44 (0.66)	1405	0.27
BL Openness to Problem Solving	1–5	3.33 (0.78)	2353	3.36 (0.75)	1376	1.29
S Openness to Problem Solving	1–5	3.39 (0.78)	1968	3.40 (0.77)	1409	0.40
BL Mathematics Interest	1–4	2.19 (0.74)	2260	2.23 (0.74)	1315	1.82
MF Mathematics Interest	1–4	2.44 (0.76)	2297	2.44 (0.75)	1411	0.15
S Mathematics Interest	1–4	2.19 (0.75)	1935	2.19 (0.75)	1390	0.06
BL Mathematics Utility	1–5	3.44 (0.87)	2357	3.50 (0.86)	1379	2.50*
S Mathematics Utility	1–4	2.83 (0.65)	1962	2.84 (0.65)	1407	0.61
Teacher Support	1–5	4.08 (0.98)	2288	4.09 (0.96)	1410	0.46
F DLEPs	1–5	3.12 (0.66)	2330	3.12 (0.65)	1423	0.15
S DLEPs	1–5	3.19 (0.73)	1923	3.19 (0.73)	1381	0.01

Note. Two-tailed *t* statistics testing mean differences between full sample (all students) and subsample (students who completed all three questionnaires). For dummy variables, proportions are presented as means, and *N* reflects all cases (0 and 1). BL = baseline; F = fall; MF = mid-fall; S = spring; DLEPs = Deep Learning and Engagement Practices.
* $p < .05$, ** $p < .01$, *** $p < .001$.

If the missingness of a variable is unrelated to both that variable and any other variable in the dataset, it is considered missing completely at random (MCAR). Most analyses with MCAR data will yield unbiased parameter estimates, as complete cases are considered as a random subsample of the complete dataset (Beaujean, 2014). In this sample, however, the observed patterns of missingness are a combination of missing at random (MAR) for some measures (i.e., a measure's missingness was unrelated to the measure itself, but related to other measures in the dataset), and not missing at random (NMAR) for others (i.e., a measure's missingness is related to that measure).

Determining whether data are missing at random or not missing at random is less an empirical consideration than a conceptual one (Beaujean, 2014); however, the distinction has important implications for the reliability of parameter estimates. Much of the observed missing data are likely MAR. As an example, many students did not report the frequency of their teacher's pedagogical practices because they were absent from school on the day the SMQ was administered. Students with regular or poor attendance may be equally likely to respond to the questionnaire items, but students with low attendance would be expected to have greater missing data on the classroom practices measures than students who attended school regularly. However, among students with similar attendance, there would be no observed relationship between missingness and reports of classroom practices frequencies. In this case, the data loss type can be considered MAR. For other measures, the MAR/NMAR distinction is less clear. Were students who believed mathematics to be not at all useful more likely to have missing data (e.g., they chose not to complete the questionnaires about mathematics because they viewed the domain as pointless)? Similarly, were students with lower mathematics skills avoiding responding to the mathematics survey items?

In order for MAR data to work properly and yield unbiased results, it is assumed that variables that explain the missingness are included (Beaujean, 2014; Kline, 2015). Four variables (fall mathematics course grade, gender, off track in math designation, and advanced math designation) were included in all structural models as a means of reducing the bias of parameter estimates while keeping the model as parsimonious as possible. In addition, all measurement and structural models employed FIML estimation. FIML does not eliminate cases with missing values from the analysis, but preserves all available data (Arbuckle, 1996), providing unbiased or less biased parameter estimates than listwise deletion (where entire case is excluded from analysis if single variable is missing) for samples with substantial missing values (Enders & Bandalos, 2001).

CHAPTER III
Introduction

In academic settings, values, competence beliefs, and goals are central antecedents of emotions (Boekaerts, 2007; Linnenbrink & Pintrich, 2002; Pekrun, 2006), and positive affective experiences can facilitate problem solving, cognitive organization, and critical thinking (Ahmed et al., 2013; Isen, 2004; Pekrun et al., 2002; Pekrun, 2006; Fredrickson, 2001). Findings from Wigfield & Cambria (2010) suggest that value may be particularly important for behaviors and behavioral intentions (e.g., course-taking choices). Expectancy-value theory considers interest as a form of valuing that refers to both interest in the subject matter and enjoyment of engagement in the activity itself (Eccles & Wigfield, 2002). As students encounter challenging learning situations, value beliefs (e.g., interest and utility of the academic subject of study) function to initiate and sustain students' engagement (Cleary & Chen, 2009; Eccles et al., 1989; Pajares, 1996). Not only is engagement strongly associated with learning (Stipek & Chiatovich, 2017), it is also instrumental in the development of academic skills (Reyes et al., 2012). Students who are engaged exhibit greater motivation to learn and greater interest (Skinner & Belmont, 1993; Fredricks et al., 2004). Disengaged students report being angry, bored, or anxious about being in the classroom (Reyes et al., 2012; Skinner & Belmont, 1993). Boredom is negatively correlated with academic performance (Pekrun et al., 2010; Pekrun et al., 2009; Daniels et al., 2009; Goetz et al., 2007), and disengaged students are more likely to have lower grades and drop out of school (Kaplan et al., 1997).

The presentation of mathematics proficiency as a broad, multidimensional construct was put forth by the NRC (2001) as a means to broaden the way successful mathematics learning is viewed and defined. Each of the five strands of mathematical proficiency (see Table 1.1 for descriptions) speaks to a different aspect of working with and learning mathematics, therefore, to be mathematically proficient requires facility in all five. The final strand, productive disposition, underscores the importance of supporting students' use of self-regulatory behaviors (e.g., perseverance) and considering the ways in which student interest in and valuation of mathematics can be advanced. Cleary and Chen (2009) examined the extent to which certain motivation variables such as mathematics task interest, perceived instrumentality of mathematics, and self-standards (i.e., the self-evaluative performance standard students set for themselves) could predict sixth- and seventh-grade students' self-regulatory strategies and maladaptive regulatory behaviors in mathematics. Their results revealed not only that interest in and perceived instrumentality of mathematics significantly predicted students' use of regulatory strategies during learning (e.g., help seeking, time management), but that all three motivational variables were negatively associated with students' use of maladaptive regulatory behaviors (e.g., forgetfulness, task avoidance). These findings are consistent with other motivation research that has underscored the importance of task interest on students' cognitive and behavioral engagement (Ahmed et al., 2013; Simpkins et al., 2006; Fredericks & Eccles, 2002). Task interest and value have been shown to be significant predictors of persistence and effort (Schunk et al., 2013; Simpkins et al., 2006), with mathematics enjoyment positively associated with the use of active problem-solving strategies, persistence at tasks, creativity, cognitive flexibility, and academic performance (Frenzel, Thrash, et al., 2007; Goetz et al., 2007; Pekrun et al., 2002; Stipek et al., 1998). An investigation conducted by Ahmed and colleagues (2013) of the developmental trajectories of grade 7 students' emotions in mathematics classrooms found that over a one-year period, enjoyment and pride decreased, boredom increased, and anxiety remained relatively unchanged. Their study also observed that changes in students' positive emotions were associated with changes in achievement and self-regulated learning strategies, corroborating findings from prior cross-sectional research (Pekrun et al., 2002; Frenzel, Thrash, et al., 2007).

Despite their established importance, students' beliefs about mathematics tend to become increasingly more negative as they progress from early grades to secondary school. Not only does students' general intrinsic motivation (i.e., enjoyment of learning) tend to decrease over time (Wigfield et al., 2006), but declines in enjoyment specific to mathematics have also been observed (Ahmed et al., 2013; Frenzel et al., 2009). A longitudinal study conducted by Fredricks and Eccles (2002) that documented changes to students' mathematics competency beliefs, mathematics interest, and valuation of mathematics from childhood to adolescence observed steady declines to students' mathematics interest, mathematics importance, and perceived ability from first through twelfth grade. A later, cohort-sequential longitudinal investigation (Watt, 2004) conducted with an Australian sample found similar declines.

Interest

A central issue in research on interest is the differing conceptualizations of the construct and the resulting highly varied approaches to its measurement. Interest has been operationalized in terms of liking (e.g., Wigfield et al., 1997; Eccles & Wigfield, 1992), value and affect (e.g., Krapp, 2002a), stored value and feelings, positive feelings, and repeated engagement (Hidi & Renninger, 2006; Renninger & Bachrach, 2015; Renninger et al., 2002). Unlike many other motivational variables, interest includes both cognitive and affective components that work as systems that are both separate and interacting (Krapp, 2000; Hidi & Harackiewicz, 2000). The development of interest involves the interaction of affect and knowledge (Hidi & Renninger, 2006; Krapp, 2002a; Renninger, 2000), and those cognitive and affective components co-occur and shift as interest develops (Renninger & Hidi, 2011). While the potential for interest lies within an individual, the environment and the content contribute to the development of interest and define its direction (Hidi & Renninger, 2006). As such, interest can be considered the outcome of an interaction between a particular context and an individual (Krapp, 2000), and its development is supported by a person's own efforts, others, and the organization of the environment.

The Four-Phase Model of Interest Development

The terms "situational interest" and "individual interest" are traditionally used to distinguish between the ways in which interest is identified and measured. Situational interest refers to the in-the-moment affective reaction and focused attention that is triggered by environmental stimuli in the situation, whereas individual interest is the predisposition of an individual to re-engage with specific content over time (Krapp, 2002b) or the psychological state at the time that predisposition has been activated (Hidi & Renninger, 2006; Renninger, 2000). Hidi and Renninger's four-phase model of interest development integrates the developmental phases of situational and individual interest into a single model. Figure 3.1 depicts key characteristics of each stage. Each phase is characterized by varying amounts of value, knowledge, and affect, and is described in terms of both cognitive and affective processes. The four interest phases are (a) triggered situational interest; (b) maintained situational interest; (c) emerging individual interest; and (d) well-developed individual interest, and are each considered as distinct and sequential phases.

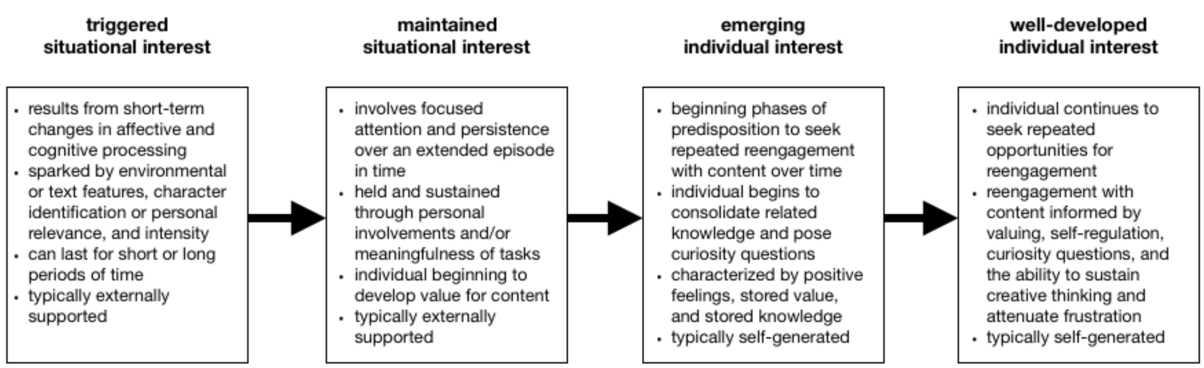

Figure 3.1. Characteristics of each interest phase of Hidi and Renninger's (2006) four-phase model of interest development.

Research suggests that the teacher, through providing students with opportunities to formulate and pursue their own learning goals, can activate situational interest in students (Dohn, 2013; Rotgans & Schmidt, 2011). Situational interest is sustained through both the meaningfulness of and one's personal involvement with tasks (Harackiewicz et al., 2000). Maintained situational interest is the phase in which a meaningful connection with the content is forged and its deeper significance is realized (Dewey, 1939). A student with well-developed interest is distinguishable by their self-regulated behavior (Harackiewicz et al., 2008; Sansone et al., 2015), such as seeking answers to questions that have aroused their curiosity (Hidi & Renninger, 2006) and persevering in the face of frustration or difficulty (Renninger, 2000), as well as by a self-driven determination to pursue deeper understanding and learn from feedback (Lipstein & Renninger, 2007). While emerging and well-developed individual interest is typically self-generated, it still requires some external support, such as environments and tasks that challenge and provide opportunity, or models such as experts and peers (Hidi & Renninger, 2006; Renninger, 2000). Hidi and Renninger assert that a well-developed individual interest enables a person to sustain long-term creative and constructive endeavors and to produce deeper levels and more forms of strategies for task engagement. A student with well-developed individual interest values the opportunity to reengage with content, will pursue such tasks when given a choice, and is likely to be resourceful when confronted with conditions in which curiosity questions cannot immediately be answered.

Person-Object Theory of Interest

As with the four-phase model of interest development, the person-object theory of interest (POI) considers interest as a specific person-object relationship (i.e., it exists in the interaction between the individual and the environment) that is content-particular and associated with positive emotions. Perception of value is considered instrumental to interest development, hypothesized to contribute both to the progression from situational interest to individual interest and to the strengthening of existing individual interest (Hidi & Renninger, 2006). While POI characterizes and defines interest by value- and feeling-related elements (Krapp, 2002a), the four-phase model discerns affect as an integral aspect of interest engagement, which, in conjunction with knowledge, informs valuing. Specifically, each phase is characterized by affect and includes some form of cognitive or knowledge processing. Moreover, value and affect are not independent from knowledge; the process of perceiving information and representing it in a manner that is valued entails developing knowledge and cognition (Hidi & Renninger, 2006; Renninger, 2000). Hidi and Renninger assert that affect can be used as an indicator of a beginning phase of interest, because an individual's knowledge may be minimal and because an affective response first triggers one's attention. The later phases of interest development (emerging and well-developed individual interest) are products of the cognitive processes that support knowledge building and the stored valuing for re-engagement. These knowledge or cognitive processing components are much more pronounced than in the earlier interest phases and more difficult to operationalize. An individual may not be metacognitively aware of their own process of interested engagement (Renninger & Bachrach, 2015); it is more typical that interest mediates the ways in which a person engages with content. Renninger and Bachrach argue that learners may not realize that their interest has been triggered, and those in later phases of interest development may pay more attention to their self-set goals or the task at hand than to their interest.

Person-object theory of interest has often been examined in terms of cognitive evaluation (e.g., self-determination theory, Deci & Ryan, 2000). The feelings of social-relatedness, autonomy, and competence – the three basic psychological needs identified in self-determination theory – are considered vital to interest development; if these needs are not met, the predisposition to re-engage with specific content of interest cannot be realized (Krapp 2002a).

The four-phase model of interest development considers the feelings of social-relatedness, autonomy, and competence to support the advancement of interest, but views the relations between deepening or developing interest and these three basic psychological needs as reciprocal (Hidi, 2000; Hidi & Renninger, 2006).

Interest and Engagement

Engagement is defined in terms of an individual's active participation or involvement and their commitment to related goals (Christenson et al., 2012), and typically references a broad range of academic and social behaviors as well as affective and cognitive experiences (Fredricks & McColskey, 2012). Interest refers both to an individual's psychological state during engagement with particular content, and to their predisposition to re-engage with that content over time (Hidi & Renninger, 2006; Renninger & Bachrach, 2015). Engagement encompasses the affective and cognitive components of interest as well as various forms of self-regulatory and participatory behaviors (Azevedo et al., 2012; Christenson et al., 2012). Thus, an individual may not have an interest in something but still be behaviorally engaged with it, but one cannot be interested without being engaged in some form (Renninger & Bachrach, 2015).

Both interest and engagement are variables that can develop in the relation or interaction between an individual and the environment (Barron, 2006; Hidi & Renninger, 2006, Krapp, 2000; Renninger & Bachrach, 2015; Reschly & Christenson, 2012). While engagement research considers interest as a motivational variable and recognizes its relevance to engagement, it has, by and large, not addressed the distinctions between phases of interest and their roles in the development of engagement. Renninger and Bachrach attribute this to such studies' focus on attributes of value and feeling and relative inattention to knowledge, a component essential to the strengthening of value and positive feelings that characterize the later interest phases (Hidi & Renninger, 2006). According to the four-phase model of interest development, interest is developed and strengthened in situations depending on the positive affect, value, and knowledge experienced with a task. It is therefore important to consider students' initial interest in the context of their prior knowledge of and experiences with an activity; students entering a situation with different levels of knowledge/experience are likely to exhibit differing degrees of initial interest.

Interest and Utility Value

Eccles (2005) identified interest value and utility value as two components of subjective task value that play a vital role in supporting student achievement and motivation. Tasks with interest value are expected to be enjoyable or fun when one is engaged in them, whereas tasks with utility value are viewed as relevant and useful beyond the immediate context or situation. Interest value and utility value have been observed to predict various motivational outcomes such as course enrollment decisions (Bong, 2001; Crombie et al., 2005; Harackiewicz et al., 2008; Meece et al., 1990) and effort (Dietrich et al., 2017; Hulleman et al., 2008; Mac Iver et al., 1991).

Initial interest and achievement goals can lead students to place utility value on educational activities (Pintrich, 2003; Wigfield & Eccles, 2002), and research has indicated a relationship between the perceived utility of a task and performance (Bong, 2001; Hulleman et al., 2008; Malka & Covington, 2005; Simons et al., 2003). Hulleman and colleagues' (2008) examination of achievement goals and task value judgments on interest and performance in a high school sports camp and a college classroom observed that in both learning contexts, interest was predicted by students' mastery goals (i.e., goals focused on increasing competence), and initial interest and task values mediating those relationships. Conley (2012) found that integrating task value and goals was more likely to predict achievement than a single motivational predictor, and Bong (2001) observed that the perceived utility value of a course predicted students' self-efficacy in that course. Hulleman et al. posit that initial interest and achievement goals increase perceptions of task value, which in turn promotes learning and the development of subsequent interest. In essence, utility value emerges from deep engagement with a task (facilitated by goals oriented towards mastery), which in turn drives the effort, attention, and persistence that drive performance.

Mathematical Experiences and Valuation Beliefs

Research that has endeavored to investigate the connections between learning contexts and students' mathematics valuation has borne valuable insights regarding approaches to address the developmental declines in mathematics value. Both engagement and external supports work to sustain and deepen interest in content (Hidi & Renninger, 2006; Renninger, 2000; Renninger & Hidi, 2002). Generally speaking, individuals choose to engage in and enjoy tasks that vary in format, are meaningful, and are moderately difficult (Stipek et al., 1998). As a content- or object-specific psychological variable (Krapp & Prenzel, 2011), interest is supported and sustained because of opportunities or challenges that an individual sees in a task, or through the efforts of others. Findings from research on interest suggest that students should experience learning contexts that promote strategy generation and problem solving, be provided with supports that enable them to experience a triggered situational interest, and be given opportunities to generate and ask curiosity questions. Dohn's (2013) investigation of task-based situational interest in grade 6 students during an engineering design program found that task novelty, autonomy, trial-and-error experimentation, and collaboration acted as triggers of students' situational interest.

Classroom organization, task features, and activity attributes can support the development of interest (Dohn, 2013; Durik & Harackiewicz, 2007; Renninger & Bachrach, 2015). Positive feelings for content, including more advanced content emphasized in high school, can be facilitated in a variety of ways that involve teacher organization of external engagement supports, such as choice in tasks (Dohn, 2013; Flowerday & Schraw, 2003), cooperative project-based learning (Blumenfeld et al., 1991; Boaler, 1998; Dohn, 2013; Renninger & Hidi, 2002), and word problems that have contexts that connect to student interests (Renninger et al., 2002). A student with a less-developed interest requires more external support from the design of the environment (e.g., activities) and/or from others (Renninger & Bachrach, 2015). Without these supports, a student may not recognize the opportunities for engagement present in a given environment (Renninger, 2010). Teacher practices and social interactions have been linked to changes in mathematics task value, self-concept, and achievement (Urdan & Schoenfelder, 2006). Instruction that facilitates the cooperation of and interaction between students can foster commitment to the class and increase student interest (Dohn, 2013; Newmann & Wehlage, 1993), and practices that provide students with some autonomy over their learning have been found to foster greater enjoyment in STEM classes (Dohn, 2013) and of mathematical tasks (Stipek et al., 1998).

While appropriate environments and responsive educators can facilitate engagement, some learning environments may constrain the possibility that interest will develop (Ainley, 2012; Turner et al., 2015). Assignments that involve repetitive computations are unlikely to foster mathematical enjoyment and engagement compared with problems that are multidimensional and connect to real life. When students perceive what they are being taught is meaningful to them and when they experience freedom in deciding their behavior, both the value they ascribe to the content being taught as well as their need for autonomy is nurtured (Wigfield et al., 2006). Promoting students' mathematics interest and emphasizing the relevance of mathematics have been found to correlate positively with students' mathematics task value and mathematics self-concept (Gaspard et al., 2015; Watt, 2004). Depending on how it is structured, small group work can foster mathematical reasoning, fluency, and conceptual understanding (Jansen, 2012).

Furthermore, the process of reasoning aloud and explaining one's thinking to peers can help a student advance their own conceptual understanding (Fuchs et al., 1997; Jansen, 2012).

The Present Study

This study examines and describes the interrelationships between select teacher practices/mathematical experiences and students' mathematics valuation (interest and utility beliefs). Figure 3.2 presents the aspects of the Mathematics Disposition Framework that are the focus of this investigation. Although research supports (see Chapter 1) the general premise that students' cognitive appraisals of their mathematical tasks/experiences produce the affects, cognitions, and beliefs relevant to the mathematics valuation aspect of Productive Disposition, it was beyond the scope of this dissertation to attempt to observe or measure that process. As such, these cognitive appraisals are presented here solely as part of the theoretical process model that underlies this research.

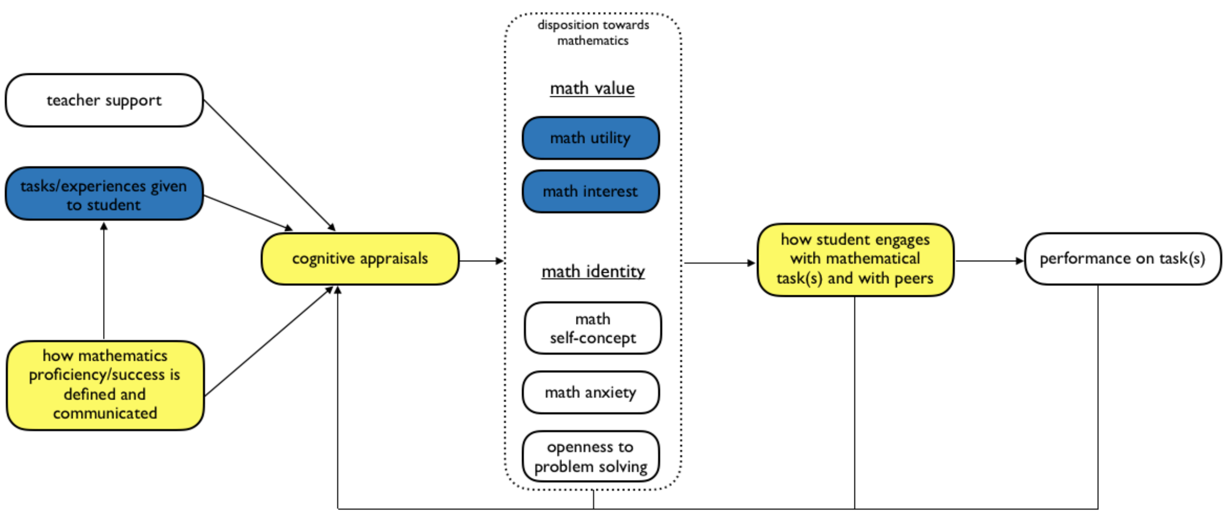

Figure 3.2. The Mathematics Disposition Framework. Color added to indicate those aspects of the framework investigated in this study: blue represents the constructs measured and of primary interest to this study, yellow represents unmeasured constructs, processes, or behaviors hypothesized to occur.

Guided by mathematics motivation literature, it was hypothesized that the data would support a directional relationship between cognitively demanding pedagogical practices and students' mathematics valuation. It was also hypothesized that three mathematical experiences (real-world connections, multi-day projects, and working in groups) would positively predict students' mathematics interest and mathematics utility, and that those experiences would mediate the relationship between the cognitively demanding practices and the two mathematics value outcomes (i.e., higher teacher use of the practices would predict greater incorporation of the three experiences, which in turn would advance students' mathematics interest and utility) .

Method

Participants and Data

This study utilized data from the Student Baseline Questionnaire (SBQ), the Student Mathematics Questionnaire (SMQ), and the Student End of Year Questionnaire (SEOY) given to the students attending the two Southern California public high schools, Ace High School and Victory High School over the course of the 2017-2018 school year. (*Note*: Participants and questionnaires are presented in more detail in Chapter 2.) Of the 1,425 students who completed the SBQ, SMQ, and SEOY, there were missing values for 513 students. Employing FIML estimation enabled the retention of 291 students with partial data in this study's analytic sample ($n = 1203$), however 222 students were eliminated from all statistical modeling due to extensive missing data.

To determine whether the data loss within this sample of 1,425 cases was random, a missing completely at random (MCAR) test that examined the equality of covariances (Jamshidian et al., 2014) was conducted. The test indicated that there was insufficient evidence to reject the hypothesis of MCAR ($p > .05$). In other words, the cases with non-missing values are assumed to represent a random sample of the 1,425 cases (Enders, 2010), and results based on only complete cases should not be biased (Kline, 2015). It is important to note that the subsample of 1,425 students who completed all three questionnaires is not representative of the full sample of students attending Ace High School and Victory High School in 2017-2018 ($n = 3285$). Chapter 2 presented a detailed comparison between the subsample and full student sample, and Chapter 5 discusses the broader limitations resulting from the missing data.

Measures

See Chapter 2 for a more robust description of all measures.

Mathematics beliefs. This study's main outcomes of mathematics interest and mathematics utility were measured using the Mathematics Utility and Mathematics Interest scales detailed in Chapter 2 (Table 2.3). Here, mathematics interest encompasses both situational interest (i.e., how interesting an individual finds a specific task or situation), and individual interest (i.e., their broad interest/enjoyment of mathematics). The Mathematics Self-Concept measure, also detailed in Chapter 2, was included in this study as a control for the two mathematics value outcomes. Students' baseline mathematics utility, interest, and self-concept were measured using SBQ data, and students' end-of-year (i.e., spring) mathematics utility and interest were measured using SEOY data.

Mathematics classroom characteristics measures. Table 2.4 presented the nine items and response options provided to students that comprise the Cognitive Activation in Math Class scale, referred to in this study as "cognitively demanding practices." This investigation utilizes the relevant SMQ items to measure the cognitively demanding practices scale for the fall semester. Additionally, three non-scale mathematical experiences – "real-world connections," "working in groups," and "multi-day projects" – are investigated in this study, using the relevant SMQ data for the fall measure and SEOY data for the spring measure of each experience. These three mathematical experiences were also presented in Table 2.4.

Mathematics course grade. District-reported mathematics course grades from the fall 2017 semester, converted to a four-point scale, were utilized.

Covariates. In addition to gender (1 = female, 0 = male) and school (1 = Ace High School, 0 = Victory High School), two other covariates were included: “advanced math” and “off track in math”. Advanced math reflects whether a student’s mathematics class was an AP/honors/accelerated course (1 = advanced course, 0 = not advanced course). Off track in math is a measure of a student’s mathematics trajectory (1 = off track, 0 = not off track).

Data Analysis

Measurement model specification. A CFA of the hypothesized multi-factor model was conducted with this study’s full analytic sample ($n = 1203$) to assess the fit of the measurement model to the observed data. Model modifications were done sparingly to minimize the likelihood of a Type 1 error of over-fitting the observed data. Latent factors were standardized to allow for free estimation of all factor loadings. To assess measurement invariance across the two schools (i.e., to examine whether the measurement model worked the same across the two groups, or whether any observed differences were due to differences in the instruments used to measure the latent variables), a multi-group measurement model was estimated by school. Including multiple groups in the model enables the researcher to examine whether or not the measurement and structural models work the same across groups (in this case, the two school sites); if group differences are observed in a model with strong invariance, it suggests that those differences are due to real differences in the latent variables, not differences in how the variables were measured (Beaujean, 2014).

Structural model specification. Figure 3.3 displays the structural model. Structural equation modeling was employed to test the direct paths from cognitively demanding practices to mathematics utility and mathematics interest, and indirect paths via three mathematical experience mediators: real-world connections, multi-day projects, and working in groups.

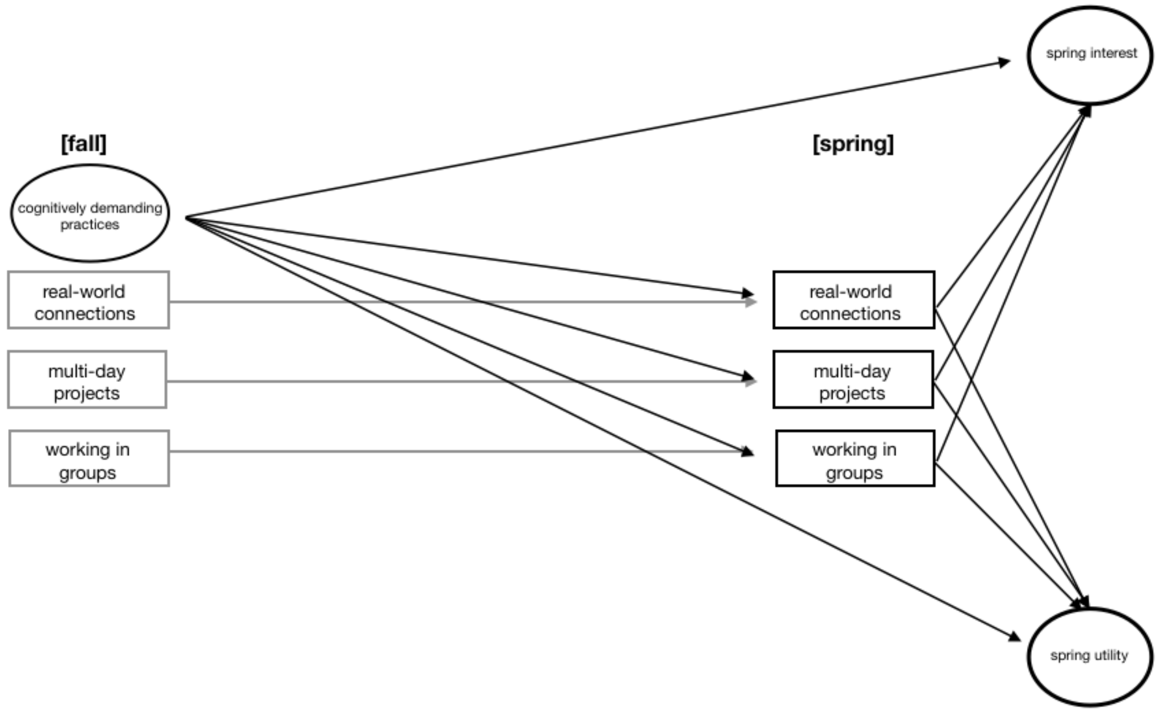

Figure 3.3. Structural pathways between practices and experiences on student mathematics value outcome variables. Covariates and baseline measures of mathematics interest and mathematics utility not pictured.

Specifically, the following estimates were calculated: (a) paths from teachers' use of fall cognitively demanding practices to students' spring mathematics interest and utility; (b) paths from teachers' use of fall cognitively demanding practices to their incorporation of real-world connections, multi-day projects, and working in groups in the spring (after accounting for the extent to which these experiences were present in the fall); (c) paths from teachers' incorporation of real-world connections, multi-day projects, and working in groups in the spring semester to students' end-of-year mathematics interest and mathematics utility; (d) paths from students' baseline mathematics interest, baseline mathematics utility, baseline mathematics self-concept, and fall mathematics course grade to students' end-of-year mathematics interest and mathematics utility (not pictured in Figure 3.3); and (e) indirect effects of all fall teacher practices on end-of-year mathematics value outcomes through teachers' *spring* incorporation of real-world connections, multi-day projects, and working in groups. This mediation analysis procedure enables indirect effects to be quantified (Hayes, 2009; Jeon et al., 2014). Indirect effects were generated via a Sobel test (Sobel, 1982) with bootstrap analysis. Bootstrapping is an iterative resampling technique that estimates statistics by drawing a high number of samples with replacement, and then generates an average across samples. Bootstrapped standard errors were computed from 5,000 bootstrap samples (the recommended minimum of resamples; Hayes, 2009).

Results

Descriptive Statistics and Correlations

Table 3.1 presents descriptive statistics for each measure. To examine the mean differences between Ace and Victory High School students, a series of two-sample *t*-tests by school were conducted. Significant differences were observed for students' mean mathematics interest (spring, greater for Victory High school), as well as for two of the classroom experiences of interest: multi-day projects (fall, greater for Ace High School), and working in groups (fall and spring, greater for Victory High School). Victory High School also had a significantly higher proportion of students enrolled in an advanced mathematics course compared with Ace High School.

Table 3.1

Descriptive Statistics

Variable	Descriptive Statistics M	SD	Range	∝	Ace M (SD)	Victory M (SD)	t
School (1 = Ace)	0.49	–	0–1	–	–	–	–
Gender (1 = female)	0.51	–	0–1	–	0.50 (–)	0.51 (–)	0.27
Off track in Math	0.24	–	0–1	–	0.26 (–)	0.22 (–)	-1.87
Advanced Math	0.23	–	0–1	–	0.19 (–)	0.27 (–)	3.92***
BL Mathematics Interest	2.23	0.74	1–4	.89	2.20 (0.73)	2.25 (0.75)	1.13
S Mathematics Interest	2.19	0.75	1–4	.89	2.15 (0.74)	2.23 (0.75)	2.00*
BL Mathematics Utility	3.49	0.87	1–5	.93	3.51 (0.86)	3.47 (0.88)	-0.87
S Mathematics Utility	2.84	0.65	1–4	.93	2.83 (0.65)	2.85 (0.65)	0.57
F Cog. Demanding Practices	2.81	0.62	1–4	.85	2.78 (0.62)	2.84 (0.61)	1.96
F Real-World Connections	3.53	1.31	1–5	–	3.55 (1.31)	3.51 (1.31)	-0.54
S Real-World Connections	3.58	1.26	1–5	–	3.55 (1.26)	3.60 (1.27)	0.72
F Multi-day Projects	2.07	1.19	1–5	–	2.16 (1.22)	1.97 (1.16)	-2.99**
S Multi-day Projects	2.61	1.33	1–5	–	2.66 (1.31)	2.57 (1.35)	-1.18
F Working in Groups	3.22	1.38	1–5	–	2.89 (1.33)	3.53 (1.35)	9.07***
S Working in Groups	3.34	1.35	1–5	–	3.12 (1.32)	3.56 (1.34)	6.08***

Note. Two-tailed t statistics testing mean differences between students attending Ace High School and students attending Victory High School. For dummy variables, proportions are presented as means. BL = baseline; F = fall; S = spring. $n = 1203$.
* $p < .05$, ** $p < .01$, *** $p < .001$.

Bivariate correlations between key measures are presented in Table 3.2. The classroom practices comprising the cognitively demanding practices measure for the fall semester were positively associated with the three spring mathematical experiences (rs ranged from .23 to .33, $p < .001$), spring mathematics interest ($r = .23$, $p < .001$), spring mathematics utility ($r = .28, p < .001$). In addition, spring real-world connections, multi-day projects, and working in groups measures were all significantly positively correlated with mathematics interest (rs ranged from .16 to .24, $p < .001$) and mathematics utility (rs ranged from .11 to .26, $p < .001$), as well as with one another (rs ranged from .41 to .45, $p < .001$). Significant positive correlations were observed between the advanced math designation and all baseline and spring belief measures (rs ranged from .16 to .18, $p < .001$), and significant negative correlations were observed between the off track in math designation and mathematics utility ($r = -.09, p <.01$, and $r = .08, p <.01$, for baseline and spring, respectively). Females reported significantly less interest in mathematics at baseline than males ($r = -.11, p < .001$), but were more likely to be taking an advanced mathematics class, and less likely to be off track in mathematics.

Table 3.2

Bivariate Correlations of Key Measures

	School	Gender	Off Track in Math	Advanced Math	BL Interest	S Interest	BL Utility	S Utility	F Cog. Dem.	S Real-World	S Projects
Gender	-0.004										
Off Track in Math	0.049	-0.085**									
Advanced Math	-0.111***	0.076**	-0.284***								
BL Interest	-0.038	-0.110***	-0.034	0.160***							
S Interest	-0.063*	-0.050	-0.036	0.159***	0.533***						
BL Utility	0.012	-0.018	-0.092***	0.182***	0.647***	0.402***					
S Utility	-0.015	0.031	-0.079**	0.167***	0.453***	0.613***	0.561***				
F Cog. Demanding Practices	-0.052	0.051	-0.041	0.237***	0.170***	0.232***	0.219***	0.276***			
S Real-World	-0.019	0.039	-0.034	0.021	0.103***	0.240***	0.141***	0.258***	0.313***		
S Projects	0.036	-0.047	0.071**	-0.013	0.076*	0.159***	0.066*	0.110***	0.225***	0.416***	
S Groups	-0.164***	0.038	-0.026	0.137***	0.114***	0.217***	0.120***	0.192***	0.326***	0.409***	0.450***

Note. Reference category for school is Victory. Reference category for gender is male. BL = baseline; F = fall; S = spring.
$* \, p < .05$, $** \, p < .01$, $*** \, p < .001$.

Measurement Model

An examination of the measurement model in which all latent variables were allowed to covary freely was conducted. Mathematics interest was indexed by four items, mathematics utility by ten items, and the cognitively demanding practices factor by nine items. The model included error covariances between individual items of a latent factor for which there was a strong theoretical justification. The measurement model had adequate fit to the data (x^2/df = 2.90, CFI = .941, TLI = .936, RMSEA = .038, and SRMR = .067).

An examination of whether the measurement model varied by school site was conducted. This was done by testing measurement invariance by school, to assess whether the model as a whole fit differently as a function of school. A series of group invariance analyses were performed with increasingly restrictive parameters (a change of less than .01 to the CFI from one model to the next implies that the invariance assumption holds; Cheung & Rensvold, 2002). Results of the invariance analyses, presented in Table 3.3, indicated strong construct reliability invariance (i.e., students reporting the same level of any given latent variable had the same expected value on that latent variable's indicator variables, irrespective of school; Beaujean, 2014).

Table 3.3

Comparison of Models Testing Measurement Invariance

Model	Chi-squared (*df*)	Comparison	Δchi-squared (*df*)	CFI	ΔCFI	RMSEA	ΔRMSEA
1. Same form model	4294.1 (1920)			.933		.041	
2. Equal loadings model	4337.4 (1955), $p < .001$	2 v. 1	43.3 (35)	.933	.000	.040	.001
3. Equal loadings and intercepts	4453.3 (1995), $p < .001$	3 v. 2	115.9 (0)	.930	.003	.041	.001
4. Equal loadings, intercepts, and residuals	4587.6 (2062), $p < .001$	4 v. 3	134.3 (67)	.929	.001	.040	.001
5. Equal loadings, intercepts, residuals, and means	4613.4 (2068), $p < .001$	5 v. 4	25.8 (6)	.928	.001	.041	.001

Note. CFI = comparative fit index; RMSEA = root-mean-square error of approximation. ΔCFI< 0.01 used to establish measurement invariance.

Structural Model

Structural equation modeling was used to assess the impact of the cognitively demanding practices and the three mathematical experiences (real-world connections, multi-day projects, and working in groups) on mathematics interest and mathematics utility over the school year. This was done by augmenting the measurement model to include structural pathways between the variables in order to test the hypotheses that the cognitively demanding practices advance student mathematics interest and mathematics utility (H1), and that these relationships are mediated by increases to teachers' incorporation of the real-world connections, multi-day projects, and working in groups experiences (H2). The structural model estimated paths between the fall latent cognitively demanding practices variable to the outcome variables of spring mathematics interest and spring mathematics utility, and mediating pathways via spring real-world connections, multi-day projects, and working in groups measures. The model included baseline mathematics interest, baseline mathematics utility, baseline mathematics self-concept, and fall mathematics course grade as controls for the two mathematics value outcomes. Additionally, pathways from the covariates of gender, school site, advanced math designation, and off track in math designation were estimated for all latent variables. Figure 3.4 presents standardized path coefficients and robust standard errors for all pathways among key variables. Table 3.4 presents path coefficients and robust standard errors for all other pathways. The structural model explained 38.8% of the observed variance in students' spring mathematics interest and 38.9% of the observed variance in students' spring mathematics utility, and fit the data adequately (x^2/df = 2.82; CFI = .933; TLI = .926; RMSEA = .037; SRMR = .065).

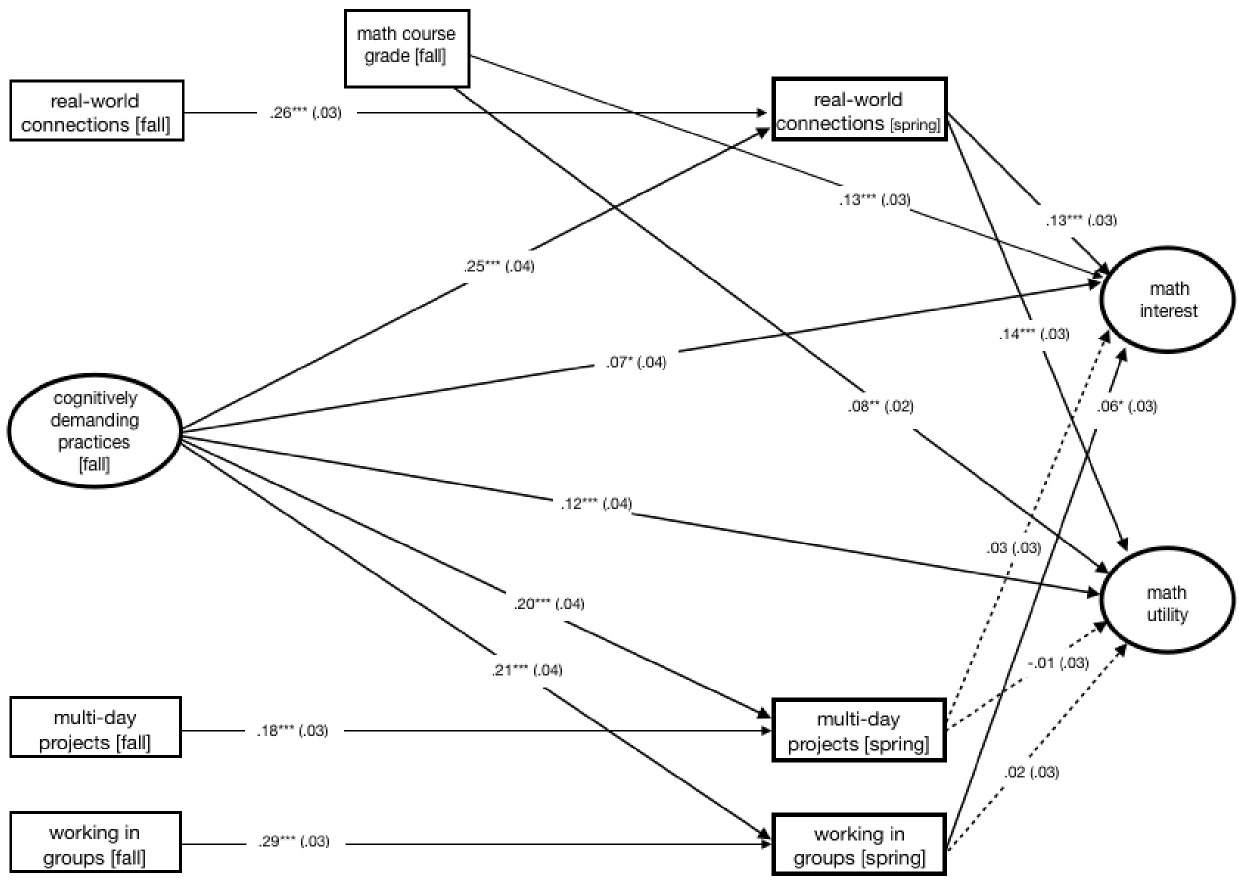

Figure 3.4. Results of path analysis. Standardized path coefficients and robust standard errors are reported. n = 1203. Controls of spring mathematics value outcome variables not pictured include school, gender, advanced math designation, off track in math designation, and baseline mathematics interest, mathematics utility, and mathematics self-concept. A dashed line indicates nonsignificant estimates.
* p < .05; ** p < .01; ***p < .001.

Table 3.4

Standardized Parameter Estimates for Model Covariates

	Baseline β (*SE*)	**Spring** β (*SE*)
Mathematics Utility		
Off Track in Math	-.06 (.07)	-.01 (.08)
Advanced Math	.18*** (.07)	-.01 (.08)
Gender	.05 (.06)	.05* (.06)
School	.04 (.06)	-.01 (.06)
BL Mathematics Utility	–	.46*** (.06)
BL Mathematics Interest	–	.12 (.09)
BL Mathematics Self-Concept	–	.03 (.07)
Mathematics Interest		
Off Track in Math	-.03 (.07)	.01 (.08)
Advanced Math	.18*** (.07)	-.01 (.08)
Gender	-.14*** (.06)	.01 (.06)
School	-.02 (.06)	-.02 (.06)
BL Mathematics Interest	–	.49*** (.09)
BL Mathematics Utility	–	.02 (.06)
BL Mathematics Self-Concept	–	.07 (.07)
Mathematics Self-Concept		
Off Track in Math	-.03 (.07)	–
Advanced Math	.32*** (.08)	–
Gender	-.16*** (.06)	–
School	-.07* (.06)	–

	Fall β (*SE*)	**Spring** β (*SE*)
Cognitively Demanding Practices		
Off Track in Math	.04 (.07)	–
Advanced Math	.25*** (.07)	–
Gender	.04 (.06)	–
School	-.04 (.06)	–
Real-World Connections		
Off Track in Math	.03 (.09)	-.03 (.08)
Advanced Math	.11*** (.08)	-.09*** (.08)
Gender	-.05 (.07)	.04 (.06)
School	.02 (.07)	-.01 (.06)
Multi-Day Projects		
Off Track in Math	.11*** (.08)	.05 (.09)
Advanced Math	.03 (.07)	-.05 (.08)
Gender	.01 (.06)	-.05 (.07)
School	.07** (.06)	.03 (.07)
Working in Groups		
Off Track in Math	.03 (.09)	.01 (.09)
Advanced Math	.23*** (.08)	-.01 (.08)
Gender	-.01 (.07)	.04 (.07)
School	-.21*** (.07)	-.08** (.07)

Note. Standardized coefficients (β) and robust standard errors (*SE*) are reported. BL = baseline. Reference category for gender is male. Reference category for school is Victory.
* $p < .05$, ** $p < .01$, *** $p < .001$.

Direct effects. The proposed paths between the latent cognitively demanding practices variable in the fall semester and the two latent mathematics outcome measures were significant. Each 1-point standard deviation (SD) increase in fall cognitively demanding practices corresponded to a .07 ($p < .05$) and .12 ($p < .001$) SD increase in spring mathematics interest and mathematics utility, respectively, after controlling for school, gender, advanced math designation, off track in math designation, fall mathematics course grade, and baseline levels of mathematics interest, utility, and self-concept. As expected, baseline mathematics interest and mathematics utility significantly predicted their respective spring interest (β = .49, $p < .001$) and utility (β = .46, $p < .001$). Among the three mathematical experiences, spring working in groups predicted significant increases to mathematics interest (β = .06, $p < .001$), and spring real-world connections predicted significant increases to both students' spring mathematics interest (β = .13, $p < .001$) and mathematics utility (β = .14, $p < .001$), after accounting for the effects of fall cognitive activation practices, fall mathematics course grade, baseline belief measures, gender, school, advanced math designation, and off track in math designation. On the other hand, after accounting for the other two experiences, spring multi-day projects was not observed to be significantly related to either value outcome.

Unsurprisingly, teachers' incorporation of real-world connections, working in groups, and multi-day projects in the fall semester significantly predicted their inclusion of real-world connections (β = .26, $p < .001$), working in groups (β = .29, $p < .001$), and multi-day projects (β = .18, $p < .001$) in the following semester. In Addition, after accounting for the extent to which teachers were already incorporating each practice in the prior semester, fall cognitively demanding practices significantly predicted teachers' incorporation of all three spring experiences (β = .25, $p < .001$, real-world connections; β = .21, $p < .001$, working in groups; β = .20, $p <$ .001, multi-day projects). Among covariates, positive significant associations were observed between gender (female) and spring mathematics utility (β = .05, $p <$ .05). Fall mathematics course grade was significantly associated with both spring mathematics interest (β = .13, $p < .001$) and spring mathematics utility (β = .08, $p <$ .01).

Indirect effects. Twelve tests of mediation using the spring mathematical experiences (real-world connections, multi-day projects, and working in groups) as mediators were conducted. Indirect effects were calculated by the products of the paths from the fall practices/experiences to the spring experiences and paths from the spring experiences to the two value outcomes. Ordinary least squares regression and the Sobel test using 5,000 bootstrap samples was used to test the significance of each indirect pathway in the model. As shown in Table 3.5, six of the hypothesized mediations were significant (indirect coefficients ranged from .01, $p <$ .05, to .04, $p <$ .001), after controlling for school, gender, advanced math designation, off track in math designation, fall mathematics course grade, baseline mathematics interest, baseline mathematics utility, and baseline mathematics self-concept.

In addition to the observed direct effects of fall cognitively demanding practices on students' spring mathematics interest and utility, there were also statistically significant indirect effects of these practices via the mediating roles of teachers' incorporation of certain mathematical experiences in the spring. Findings revealed that a 1 SD increase to teachers' implementation of cognitively demanding practices in the fall semester predicted an additional .04 SD increase to students' mathematics interest and mathematics utility at the end of the year. Specifically, students' exposure to real-world connections in the spring semester mediated the observed relations between (a) fall cognitively demanding practices and spring mathematics interest (β = .03, $p <$ .001), and (b) fall cognitively demanding practices and spring mathematics utility (β = .04, $p < .001$), and students' spring semester exposure to working in groups mediated the effect of fall cognitively demanding practices on spring mathematics interest (β = .01, $p < .05$). In other words, students experiencing more frequent cognitively demanding practices in their fall semester mathematics courses went on to report higher mathematics interest and mathematics utility at the end of the school year, and these increases can be explained in part by their teachers' incorporation of real-world connections and group work into their spring semester lessons.

Table 3.5

Indirect Effects

	β (*SE*)
Real-World Connections	
F Cognitively Demanding Practices → S Real-World Connections → S Mathematics Utility	.04*** (.01)
F Cognitively Demanding Practices → S Real-World Connections → S Mathematics Interest	.03*** (.01)
F Real-World Connections → S Real-World Connections → S Mathematics Utility	.04*** (.01)
F Real-World Connections → S Real-World Connections → S Mathematics Interest	.03*** (.01)
Multi-Day Projects	
F Cognitively Demanding Practices → S Multi-Day Projects → S Mathematics Utility	-.00 (.01)
F Cognitively Demanding Practices → S Multi-Day Projects → S Mathematics Interest	.01 (.01)
F Multi-Day Projects → S Multi-Day Projects → S Mathematics Utility	-.00 (.01)
F Multi-Day Projects → S Multi-Day Projects → S Mathematics Interest	.01 (.01)
Working in Groups	
F Cognitively Demanding Practices → S Working in Groups → S Mathematics Utility	.01 (.01)
F Cognitively Demanding Practices → S Working in Groups → S Mathematics Interest	.01* (.01)
F Working in Groups → S Working in Groups → S Mathematics Utility	.01 (.01)
F Working in Groups → S Working in Groups → S Mathematics Interest	.02* (.01)

Note. Indirect standardized coefficients (β) through mediators and bootstrap standard errors (*SE*) from 5,000 bootstrap samples are reported. F = fall; S = spring.
* $p < .05$, ** $p < .01$, *** $p < .001$.

Table 3.5 also shows that, after controlling for fall cognitively demanding practices, fall real-world connections was significantly associated with mathematics valuation outcomes through spring real-world connections, indicating that teachers who incorporated more real-world connections in the fall were more likely to do so in the spring, which in turn predicted increases to mathematics interest (β = .03, $p < .001$) and mathematics utility (β = .04, $p < .001$). Similarly, students reporting more group work in the fall semester were more likely to experience group work in the spring, which subsequently predicted significant increases to mathematics interest ($\beta = .02, p < .05$).

Discussion

This study's findings revealed that, after accounting for fall mathematics course grade, baseline mathematics interest, utility, and self-concept, and covariates (gender, school, advanced math, off track in math), students who experienced more cognitively demanding practices in their mathematics courses found mathematics to be more interesting and useful at the end of the year, and that these advances can in part be attributed to students' exposure to learning contexts that were collaborative and connected mathematical content to the real world.

Progress across interest phases requires positive feelings and opportunities for students to build knowledge; as content knowledge develops, students' feelings about and the valuing of content are heightened (Renninger & Balrach, 2015). A classroom climate that provides students with authentic and meaningful experiences with mathematics can give them opportunities to connect their interests and personal goals to their learning experiences, support the task value they attach to the course content, and foster their feelings of autonomy and competence (Gentry & Owen, 2004; Wang, 2012). Providing students with frequent opportunities to work collaboratively and apply their mathematical reasoning and problem-solving skills to real-world contexts, such as with project-based learning and cooperative group work, can advance students' valuation of mathematics (Hidi & Renninger, 2006; Hoffmann, 2002; Newman & Wehlage, 1993). Students who work collaboratively in groups are more comfortable conversing about mathematical content and challenging each other's ideas, and are more likely to take responsibility for shared understanding (Gresalfi, 2009; Webb & Mastergeorge, 2003). Furthermore, students that demonstrate a greater ability to analyze and critique the work of their peers have been found to report greater productive dispositions to mathematics (Gilbert, 2014), as evidenced by fewer negative emotions and higher task values and mastery goals.

H1, expecting direct effects of cognitively demanding practices on the two value outcomes, was supported for both mathematics interest and mathematics utility. This adds to the growing body of research that connects interest to learning environments that delegate autonomy to students, allow for trial-and-error experimentation (e.g., Dohn, 2013), challenge students (e.g., Hidi & Renninger, 2006), and provide opportunities for knowledge-building (e.g., Renninger & Balrach, 2015). H2, expecting indirect effects of cognitively demanding practices through real-world connections, multi-day projects, and working in groups, was partially supported for real-world connections and working in groups. Real-world connections and cooperative, project-based learning have been shown to increase positive feelings for mathematics (Blumenfeld, et al., 1991; Boaler, 1998; Gilbert, 2014; Renninger et al., 2002; Renninger & Hidi, 2002). In this study, these associations were more strongly supported for real-world connections and working in groups than for multi-day projects. The lack of observed significant relationships between multi-day projects and mathematics interest or mathematics utility may be attributed to a degree of multicollinearity between the multi-day projects measure and the other two spring experience variables. (Not only were moderate, positive correlations observed among the three experiences, but, when tested individually, the multi-day projects variable was observed to be significantly related to both value outcomes.) These findings should not be interpreted as evidence that project-based learning experiences do not advance students' mathematics interest and mathematics utility; rather, that any potential effects of multi-day projects on students' mathematics interest and mathematics utility may be explained by students' engagement with tasks that connect to the real-world and their collaboration with one another that may be happening as they work on projects. In other words, when considered simultaneously alongside two other experiences that are often characteristic of problem-based learning situations (i.e., tasks that connect to real-world contexts and opportunities for students to work collaboratively), the discernable impact of multi-day projects on students' mathematics valuation is likely more difficult to observe. It was outside the scope of this investigation to assess the extent to which project-based learning impacts students' mathematics valuation via the opportunities it affords for students to work together and apply their learning to real-world contexts. Continued research is needed to advance understandings of those relationships.

Conclusion

It is important to consider not only the opportunities certain teaching practices afford to students, but also the extent to which students are required to engage with the mathematics and one another. As mathematician Ruben Hersh (1997, cited in Boaler, 2015a) put it:

> The mystery of how mathematics grows is in part caused by looking at mathematics as answers without questions. That mistake is made only by people who have had no contact with mathematical life. It's the questions that drive mathematics. Solving problems and making up new ones is the essence of mathematical life. If mathematics is conceived apart from mathematical life, of course it seems – dead. (p. 27)

The cognitively demanding practices variable used in this study encapsulates teacher practices that are inquiry-based (e.g., "*the teacher presents problems for which there is no immediately obvious method of solution*" [CA9]), student-centered (e.g., "*the teacher asks us to decide on our own procedures for solving complex problems*" [CA3]), and emphasize the multidimensionality of mathematics (e.g., "*the teacher gives problems that can be solved in several different ways*" [CA4]). Traditional mathematics instruction tends to center on the teaching of prescribed operations and procedures to students in order to solve problems; students are taught content that often appears as a series of answers to questions that nobody asked

(Boaler, 2015a). In these classrooms, the teacher is in control of student learning, and correct solutions are emphasized (Turner et al., 2011). These procedurally focused experiences (i.e., the approach of students learning a procedure taught by a teacher, to then be repeated) not only reinforce to students that the domain of mathematics has already been decided and simply needs to be memorized (Boaler, 2015a), but limit students' ability to grasp the connected nature of mathematics concepts (Letwinsky & Cavender, 2018), an instrumental component in forming deeper understandings. That said, incorporating mathematical tasks rich with opportunity for students to actively participate and reason about the content does not guarantee that students will not resort to looking for answers if the teacher fails to hold students accountable for how they engage with the activity (Furtak, 2006; Gresalfi et al., 2012). For example, although group work can facilitate students' discourse about mathematics content and collaboration with one another, *how* students experience group work can vary based on a variety of factors. The degree to which collaborative learning is productive depends on the nature of the task, how students are taught to engage with one another, and whether and how the teacher transferred responsibility to students. The mathematics classroom includes patterns of interaction, attitudes, understandings, norms, and assumptions that function to structure activity (Cobb et al., 2009; Gresalfi, 2009), and learners' characteristic manners of engaging with the mathematical content are the result of complex interactions between themselves, others, and the material in a particular activity system that determines what learners are capable and willing to do together (Gresalfi, 2009). The relationships or identities that students develop in relation to their participation in an activity depends heavily on the affordances and features of the activity (Boaler & Greeno, 2000; Cobb et al., 2009; Hand & Gresalfi, 2015). Study 2 examines the associations between such affordances and features provided by the classroom context and the identity aspects of students' dispositions towards mathematics.

CHAPTER IV
Introduction

Compared with other subject areas, the developmental decline to academic intrinsic motivation is the most severe for mathematics (Gottfried et al., 2007). The NCTM identified "[increasing] the number of high school graduates, especially those from traditionally underrepresented groups, who are interested in, and prepared for, STEM careers" (2014, p.3) as a persistent challenge. Prior longitudinal studies that have investigated mathematics motivation variables have found that motivation tends to decrease as students progress across grade levels, particularly with respect to mathematics self-concept, utility, and interest (Fredricks & Eccles, 2002; Jacobs et al., 2002; Nagy et al., 2010; Petersen & Hyde, 2017; Watt, 2004). Individual differences in motivation are predictive of both short- and long-term achievement success and failure (Dweck, 1986; Gottfried et al., 2007; Molden & Dweck, 2006). As content becomes more challenging, students may conclude that they are not interested in learning mathematics or that it is not important as a means of protecting their self-esteem (Fredricks & Eccles, 2002). The development of mathematics skills and mathematics interest is interrelated; students with stronger mathematics skills become more interested in mathematics, and students who become more interested in mathematics acquire better skills (Fisher et al., 2012; Jõgi et al., 2015). Mathematics achievement is thus a significant contributor to changes in mathematics motivation.

Beliefs and Self-Regulatory Behaviors

The NCTM (2014) asserts that mathematics education is "driven by a nonnegotiable belief that we must develop mathematical understanding and *self-confidence* [emphasis added] in all students." According to Stipek et al. (1998), self-confidence in mathematics has important implications for student behavior in mathematics achievement contexts, such as the willingness to grapple with difficult tasks, and underlies other important motivation constructs such as expectations for success and failure in mathematics. Not only is mathematics self-concept a strong predictor of achievement (Crombie et al., 2005; Denissen et al., 2007; Marsh et al., 2005; Meece et al., 1990; Watt et al., 2012), it is critical to sustained mathematics engagement (Priess-Groben & Hyde, 2017).

Mathematics anxiety is considered as a distinct construct (Ashcraft & Ridley, 2005; Tobias, 1995) that refers to a state of discomfort in response to performing mathematical tasks (Ma & Xu, 2004). It has an affective component, which concerns tension or nervousness associated with negative psychological reactions felt in evaluative contexts, and a cognitive component, which describes concerns about performance (Dowker et al., 2016; Wigfield & Meece, 1988). Mathematics anxiety has been found to negatively correlate with focus on tasks and mathematics performance (Ahmed et al., 2013; Carey et al., 2016; Cates & Rhymer, 2003; Eynde et al., 2006; Hembree, 1990; Ma, 1999). Students with mathematics anxiety are more likely to develop negative attitudes toward the discipline and demonstrate avoidance behaviors such as dropping out of mathematics courses or avoiding mathematics-related educational tracks (Ashcraft, 2002; Ma, 1999). Mathematics competence beliefs (e.g., self-concept, self-efficacy) are considered among the strongest predictors of mathematics anxiety (Hembree, 1990; Meece et al., 1990; Parajes & Miller, 1994). Many socio-cognitive models such as socio-cognitive theory (Bandura, 1997) and control-value theory (Pekrun, 2006) consider mathematics self-concept and mathematics anxiety to have a reciprocal relationship. That is, as students experience higher levels of mathematics anxiety, their perceptions of mathematical competence fall, but as self-concept rises, students experience lower levels of anxiety. Extensive support exists for this theory (Ahmed et al., 2012; Frenzel, Pekrun, & Goetz, 2007; Goetz et al., 2006; Meece et al., 1990).

Theoretical self-regulation models postulate that one's motivational beliefs impact the degree to which one chooses to employ self-regulatory strategies during learning (Eccles

& Wigfield, 2002; Zimmerman, 2000). The expectancy-value framework (Eccles, 1994; Eccles & Wigfield, 2002; Wigfield & Eccles, 2000) emphasizes the motivational role of individuals' perceptions, interpretations, and beliefs on behavior. Strong determinants of why students would want to become or remain engaged in a mathematical task are their values (of the task, domain) and their expectancies of success (Anderman & Wolters, 2006). According to Bandura's (1997) social cognitive theory, interests in activities are formed when individuals view themselves as capable and anticipate positive outcomes. Anxiety is likely to occur in situations where the situational challenge is greater than an individual's perceived capability, whereas boredom tends to result when the situational challenge is lower an individual's perceived capability (Csikszentmihalyi, 1997). Bandura (1986) argues that it is beliefs that determine what people do with the skills and knowledge they have, and thus, behavior is better predicted by people's beliefs than by their actual capabilities.

Implicit Theories of Intelligence

Implicit theories of intelligence (Dweck & Leggett, 1988; Dweck & Molden, 2005) refer to the fundamental underlying beliefs that individuals hold about their abilities or intelligence regarding whether or not they can change. The belief that mathematics ability is a fixed, innate characteristic reflects an *entity* theory of intelligence, more commonly referred to as a fixed mindset. In contrast, the belief that mathematics ability is malleable and therefore capable of being developed reflects an *incremental* theory of intelligence (also known as a growth mindset). Research conducted by Dweck and colleagues over the last four decades indicates that individuals' implicit theories of intelligence provide a cognitive framework that orients their behavior, emotions, and cognitions in achievement situations (Blackwell et al., 2007; Burnette et al., 2020; Dweck & Leggett, 1988; Dweck & Molden, 2005; Molden & Dweck, 2006).

Research on theories of intelligence suggests that the ways individuals frame intelligence have differential effects on emotions, behavior, and cognitions in achievement contexts. For example, individuals who hold an entity theory regarding their mathematics ability often view the need to exert effort on a task as evidence of a lack of ability, as opposed to those that hold an incremental theory and see effort as a necessary component of developing their ability. Findings from research on students' implicit theories of mathematics ability (e.g., Dweck, 2012; Dweck & Leggett, 1988; Shively & Ryan, 2013; Smiley et al., 2016; Yeager & Dweck, 2012; Yeager et al., 2014) signal the importance of a growth mindset in sustaining student motivation in mathematics. A fixed ability mindset can diminish student's willingness to engage with mathematics and persevere when challenged. Beliefs that intelligence is unchangeable can lead students to interpret academic failure as evidence that they are unintelligent, which can weaken their resilience in the face of academic challenges (Blackwell et al., 2007; Yeager & Dweck, 2012).

Role of Mathematical Experiences

The way that a student participates in an activity or set of activities associated with a domain contributes to their identity concerning that domain (Barton et al., 2013; Hand & Gresalfi, 2015; Leander et al., 2010). A student's participation in a mathematics classroom can therefore create a momentum toward or away from an identity as someone who does mathematics. The classroom culture shapes the ways that learners are expected to engage with the course material and with one another (Wortham, 2004). The interaction between the nature of one's engagement with mathematics and particular content is a critical component of what one comes to know and who they come to be (Gresalfi, 2009). The identities and other beliefs that students develop while participating in an activity are connected to the affordances and features of the activity (Boaler & Greeno, 2000; Cobb et al., 2009; Hand & Gresalfi, 2015). For example, there is a relationship between the types of problems or tasks given to students and the types of ideas about the discipline that students are likely to form (Schoenfeld, 1988). Thus, a class culture that emphasizes completing several problems quickly can reinforce a binary view of mathematics: that it is either understandable or not (e.g., "if you cannot do this quickly, you do not grasp the material"). Boaler and Greeno (2000) found that students who were required to memorize and recall information given to them were either highly identified or highly disidentified with mathematics, whereas those asked to draw from their own interpretations of mathematics to make sense of procedures and concepts had multifaceted and generally more positive relationships with the domain.

Classroom norms, routines, and teachers' practices contribute to students' perceptions of what constitutes success and the purposes for engaging in academic tasks (Patrick et al., 2011). Teachers therefore play a critical role in students' valuation of academic domains and their self-concepts of ability (Diemer et al., 2016; Eccles, 1994). Different classroom practices provide students with varied opportunities for students to construct knowledge, learn to work together, and advance positive perceptions of themselves as learners and doers of mathematics. If there are limited opportunities for students to evaluate each other's ideas and justifications, or to pose questions themselves, students are more likely to engage with mathematics in a limited way. In a classroom where mathematical identities are based solely on correctness rather than sense-making, if students are unsure whether their contributions are correct, they are less likely to share their mathematical reasoning or ideas (NCTM, 2018). Deep engagement with an activity is facilitated by practices that include opportunities for self-expression, afford access to and transparency of the domain, and position students as essential to the accomplishment of collaborative goals (Barton et al., 2013; Boaler & Staples, 2008). In contrast, practices that provide marginal roles for students and limited access to the domain are less likely to support the negotiation of productive learning identities (Hand & Gresalfi, 2015; Nasir & Hand, 2008).

If students' mathematics competency beliefs are predominantly based upon comparisons of their performance with that of their peers, they are more likely to engage in maladaptive motivational behaviors such as poor persistence and weak effort (Cleary & Chen, 2009; Zimmerman, 2000). Large-scale observational studies (e.g., Remillard & Bryans, 2004; Tarr et al., 2013) have established the prevalence of teacher-centered pedagogical practices in mathematics classrooms, characterized by low-level questioning, inadequate opportunity for student discussion, and a prioritization on covering content. Compared to elementary schools, secondary school classrooms tend to be more performance-oriented, with greater emphasis on course grades and normative comparisons, as well as less autonomy for students during activities (Urdan & Midgley, 2003). Students are more likely to exhibit a fixed mindset view of mathematical competence when their teacher focuses on answers over strategies and provides content help that lowers the cognitive demand on students (Jansen, 2012). Students are less interested in the content and less motivated when there is an absence of challenging tasks or when the classroom discourse is restricted to closed questions that do not provide students with the opportunity to express their thoughts or ideas (Hannula, 2006). Instructional practices that focus students on developing understanding and learning rather than on their performance in relation to their peers foster feelings of competence (NCTM, 2018; Stipek et al., 1998).

Student engagement with tasks that focus on mathematical connections, problem solving, and reasoning has been found to support student achievement in mathematics (Boston & Smith, 2009; Stein & Smith, 2011; Sullivan et al., 2015). Students' attitudes and beliefs about themselves as learners become more positive when their strategic competence in solving unconventional problems advances. Engagement with open-ended, multidimensional problems supports the development of a growth mindset; students are more likely to believe that mathematical competence is malleable and to focus their learning on content understanding over task completion in small-group learning contexts in which the teacher transfers responsibility to students, encourages multiple solution strategies, and presses for conceptual understanding (Blad, 2015; Boaler, 2015b; Jansen, 2012). Tasks that can be approached and completed with a variety of methods offer students greater opportunity to demonstrate competence and ultimately view themselves as mathematically competent (Boaler & Staples, 2008). Opportunities for students to engage deeply with the mathematical content are supported via teacher practices that value students' willingness to take risks in sharing ideas and openness to suggestions and feedback to their ideas (Gresalfi, 2009; Lampert, 1990). For example, by highlighting student ideas or approaches, teachers can support collaboration and demonstrate to students that they are capable of making valuable contributions.

Classroom Climate

According to self-determination theoretical frameworks, a classroom climate is optimized when the learning context is perceived by students to fulfill their needs for autonomy, competence, and relatedness (Deci & Ryan, 2000). The classroom emotional climate is created by the quality of emotional and social interactions in the classroom among and between the students and teachers (Reyes et al., 2012), and these interactions can either promote or corrode student motivation and achievement (Diemer et al., 2016; Eccles & Roeser, 2011). Both students' social-emotional and academic outcomes are influenced by classroom climate (Pianta & Hamre, 2009). Engagement and academic motivation research has identified authentic instruction, autonomy support, collaboration promotion, and teacher social support as aspects of classroom climate that are central to students' achievement in mathematics (Cleary & Chen, 2009; Patrick et al., 2011; Wigfield et al., 2006). The classroom climate is optimized when it is able to support both the value students assign to the content and their confidence in their capability to master it (Eccles et al., 1993). Research has documented significant associations between social-emotional climate to student engagement (Patrick et al., 2011; Reyes et al., 2012; Stipek & Chiatovich, 2017) and a variety of other learning outcomes. A positive classroom climate is associated with less disruptive student behavior and increased academic performance, school satisfaction, and mastery motivation (Baker, 1999; Barth et al., 2004). Teacher support has been identified as an important correlate of student engagement, motivation, and achievement in classroom climate research (Patrick et al., 2011). Students in classrooms that are emotionally supportive report higher engagement, interest, and enjoyment (Curby et al., 2009; Fraser & Fisher, 1982; Skinner & Belmont, 1993), choose more complex cognitive activities (Howes & Smith, 1995), and accomplish more academically (LaRocque, 2008; Rimm-Kaufman & Chiu, 2007).

The Current Study

Figure 4.1 highlights the aspects of the Mathematics Disposition Framework that guide this investigation. While this study does not explicitly measure the ways in which mathematics success is being defined and presented to students, research on achievement goals and classroom structures indicate a positive correlation between classrooms with high mastery goal structures and the mathematics classroom characteristics measures present in this study, namely those that compose the Deep Learning and Engagement Practices (DLEPs) and teacher support factors presented in Chapter 2. Research on students' goal orientations has suggested that a performance-goal structure is fostered through practices that make differences in ability salient, such as homogenous and fixed groupings, evaluations communicated in terms of students' relative performance, uniform assignment of tasks, and rigid time structures (Ames, 1992; Meece et al., 2006). A learning/mastery orientation, in contrast, is fostered in settings where inadequate solutions and errors are treated as part of the process and as a helpful part of learning (Patrick et al., 2011; Stipek, 1998; Stipek et al., 1998). Teachers' decisions about assessment and evaluation procedures contribute to students' endorsement of mastery-approach goals (Schweinle et al., 2006; Stipek et al., 1998, Turner & Patrick, 2004). Teachers weaken mastery-approach goals by choosing to focus their grading of student work primarily on correctness of answers rather than on mathematical reasoning or solution approach; they strengthen mastery-approach goals by emphasizing approaches and explanations, encouraging students to critique the reasoning of their peers, and providing mathematical tasks that require persistence (Gilbert, 2014).

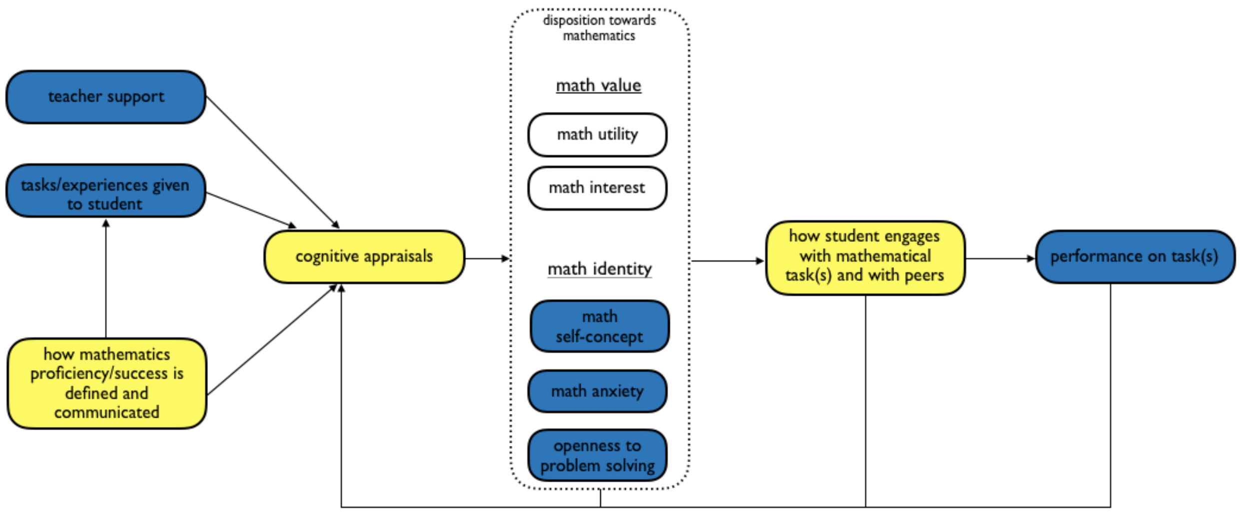

Figure 4.1. The Mathematics Disposition Framework. Color added to indicate the aspects of the framework investigated in this study: blue represents the constructs measured and of primary interest to this study, yellow represents unmeasured constructs, processes, or behaviors hypothesized to occur. A higher-order factor of mathematics comfort was used in the structural models instead of the mathematics self-concept and mathematics anxiety factors.

There is also considerable shared variance between measures of a mastery goal structure and those of positive classroom social-emotional environments featuring high levels of mutual respect and teacher support (Patrick et al., 2011; Patrick et al., 2003; Turner et al., 2002). Consistent positive associations have been found between classroom mastery goal structure and self-efficacy, effort, use of effective learning strategies, achievement, positive school-related emotion, and satisfaction with learning (see Kaplan & Maehr, 2007 for a review). Differences in teacher-student relationships between high and low mastery-focused classes have also been found; in particular, teacher support and respect are observed to be stronger in classrooms with a high mastery goal structure (Patrick et al., 2011).

Research Objective

The primary research objective of this study was to investigate the relationships between mathematics classroom experiences and mathematics identity beliefs. Specifically, this study aimed to identify (a) the impact of two aspects of students' mathematics learning context (DLEPs and teacher support) on their mathematics identity beliefs (mathematics comfort and openness to problem solving), and (b) the extent to which those relationships were mediated by students' valuation of mathematics (measured by their mathematics interest) and performance (measured by their mathematics course grade).

It was hypothesized that students who more frequently experienced the DLEPs in their mathematics courses and who considered their teachers to be more supportive of students' learning would report greater openness to problem solving and comfort with mathematics at the end of the academic year, and that those increases could be attributed to increases to mathematics interest and course performance (which, in turn, advance students' mathematics identity beliefs).

Method

Data

Three questionnaires – the SBQ, the SMQ, and the SEOY – were used to measure students' mathematics identity beliefs and the frequency of certain classroom practices and experiences taking place in their mathematics courses. (See Chapter 2 for more details on the schools, participants, data collection procedures, and questionnaires.) A total of 1,425 students, 702 from Ace High School and 723 from Victory High School, completed the three questionnaires, 1,139 of which were included in this study's analytic sample (286 cases with partial data were dropped because they were missing values for key variables that precluded their retention with FIML estimation). An analysis of the pattern of missing data indicated that the data loss could be considered random (i.e., there was insufficient evidence to reject the missing completely at random (MCAR) hypothesis, $p > .05$), and the 1,139 cases reflects a random sample of the 1,425 students who completed all three surveys.

Table 4.1 presents descriptive statistics by school. For latent variables, the mean was calculated from indicator items and is presented along with the Cronbach's alpha. Independent sample t-tests revealed no significant differences between the two schools on gender, fall semester mathematics course grade, teacher support, baseline openness to problem solving, mathematics interest (baseline and mid-fall), and the proportion of students off track in their mathematics course trajectory, $ps > .05$. However, compared with Ace High School, Victory High School had a higher proportion of students enrolled in an advanced mathematics course, a greater frequency of DLEPs occurring in its mathematics courses in the fall and spring semesters. Additionally, the students at Victory High School reported more overall mathematics comfort (baseline and spring) and openness to problem solving (spring) compared to those at Ace High School.

Table 4.1

Descriptive Statistics

	Descriptive statistics				Ace	Victory	
Variable	*M*	*SD*	Range	∝	*M* (*SD*)	*M* (*SD*)	*t*
School (1 = Ace)	0.49	–	0–1	–	–	–	–
Gender (1 = female)	0.51	–	0–1	–	0.50 (–)	0.51 (–)	0.27
Off Track in Math	0.24	–	0–1	–	0.26 (–)	0.22 (–)	-1.87
Advanced Math	0.23	–	0–1	–	0.19 (–)	0.27 (–)	3.92***
BL Mathematics Comfort	2.45	0.66	1–4	.90	2.39 (0.64)	2.50 (0.67)	3.13**
S Mathematics Comfort	2.44	0.66	1–4	.89	2.35 (0.65)	2.54 (0.66)	5.51***
BL Openness to Problem Solving	3.35	0.76	1–5	.79	3.35 (0.75)	3.35 (0.76)	-0.14
S Openness to Problem Solving	3.40	0.77	1–5	.81	3.32 (0.77)	3.47 (0.77)	3.57***
BL Mathematics Interest	2.23	0.74	1–4	.89	2.20 (0.73)	2.25 (0.75)	1.13
MF Mathematics Interest	2.44	0.75	1–4	.88	2.42 (0.73)	2.46 (0.78)	0.92
Teacher Support	4.09	0.96	1–5	.89	4.09 (0.96)	4.09 (0.96)	0.09
F DLEPs	3.13	0.65	1–5	.89	3.08 (0.66)	3.17 (0.64)	2.80**
S DLEPs	3.19	0.73	1–5	.92	3.11 (0.72)	3.27 (0.73)	3.94***
F Mathematics Course Grade	2.23	1.32	0–4.3	–	2.21 (1.32)	2.25 (1.32)	0.60

Note. Two-tailed *t* statistics testing mean differences between students attending Ace High School and students attending Victory High School. For dummy variables, proportions are presented as means. BL = baseline; F = fall; MF = mid-fall; S = spring; DLEPs = Deep Learning and Engagement Practices. * $p < .05$, ** $p < .01$, *** $p < .001$.

Measures

Several variables were included in this study to examine the relationship between students' classroom experiences and their mathematics identity beliefs. Bivariate correlations among latent and manifest variables are presented in Table 4.4 at the end of this section.

Mathematics comfort. Mathematics self-concept and mathematics anxiety are empirically and conceptually distinct constructs (Ahmed et al., 2012; Lee, 2009). However, the strong, negative correlations between the two measures, observed here and in prior research (e.g., Ahmed et al., 2012; Hembree, 1990; Pajares & Miller, 1994), makes the simultaneous inclusion of both mathematics self-concept and mathematics anxiety in the structural models difficult due to issues of multicollinearity. A higher-order latent variable, Mathematics Comfort, was therefore created for use in this study using the Mathematics Anxiety and Mathematics Self-Concept items measured on the SBQ and the SEOY. More specific details of this measure were provided in Chapter 2. Table 4.2 presents the individual items for Mathematics Comfort along with the means and standard deviations for each item on the SBQ and SEOY.

Table 4.2

Mathematics Comfort Scale Items

Mathematics Comfort			**Thinking about studying mathematics, to what extent do you agree with the following statements?**
	SMQ M (SD)	SEOY M (SD)	**1 (strongly disagree), 2 (disagree), 3 (agree), 4 (strongly agree)**
SC1	2.5 (.94)	2.5 (.96)	*I am just not good at mathematics*
SC2	2.8 (.80)	2.7 (.88)	*I get good grades in mathematics.*
SC3	2.5 (.87)	2.5 (.87)	*I learn mathematics quickly.*
SC4	2.3 (1.01)	2.3 (1.01)	*I have always believed that mathematics is one of my best subjects.*
SC5	2.3 (.87)	2.3 (.89)	*In my mathematics class, I understand even the most difficult work.*
ANX1	2.9 (.87)	2.8 (.89)	*I often worry that it will be difficult for me in mathematics classes.*
ANX2	2.5 (.87)	2.5 (.91)	*I get very tense when I have to do mathematics homework.*
ANX3	2.4 (.88)	2.4 (.87)	*I get very nervous doing mathematics problems.*
ANX4	2.2 (.88)	2.3 (.92)	*I feel helpless when doing a mathematics problem.*
ANX5	2.8 (1.00)	2.8 (1.02)	*I worry that I will get poor grades in mathematics.*

Note. Scores from SC1 and ANX1-5 were inverted.

Openness to problem solving. On the SBQ and SEOY, students were presented with a series of statements considered to indicate an openness to problem solving ("*I can handle a lot of information*," "*I am quick to understand things*," "*I seek explanations for things*," "*I can easily link facts together*," and "*I like to solve complex problems*,") and asked to what extent each statement described themselves. Responses were recorded on a 5-point Likert scale (1 = not at all like me, 5 = very much like me).

Mathematics interest. The Mathematics Interest scale was included on the SBQ, SMQ, and SEOY. Students were asked to report the extent to which they agreed with four statements about their interest in mathematics ("*I enjoy reading about mathematics*," "*I look forward to my mathematics lessons*," "*I do mathematics because I enjoy it*," "*I am interested in the things I learn in mathematics*"). The response options ranged from strongly disagree to strongly agree, and were recorded on a 4-point Likert scale (1 = strongly disagree, 4 = strongly agree).

Classroom experiences. Students were asked to report the frequency of their mathematics teacher's use of specific practices that serve as indicators for the latent Deep Learning and Engagement Practices and Teacher Support variables used in this study. Table 4.3 presents each item along with the means and standard deviations.

The DLEPs factor is defined by 15 teacher practice indicators. It measures the teachers' incorporation of real-world and collaborative learning opportunities, use of cognitively demanding practices, promotion of higher-order thinking skills, and facilitation of student reasoning. DLEPs are a composition of the 2012 NSSME Reform-Oriented Practices scale, the PISA 2012 Cognitive Activation in Math Class scale, and two individual PISA 2012 mathematical experiences items. (More details of the DLEPs scale provided in Chapter 2.) DLEPs were measured for the fall semester on the SMQ and for the spring semester on the SEOY. The PISA 2012 Teacher Support scale was included on the SMQ. Teacher support is a class climate variable that measures a teacher's regard for their students' perspectives and responsiveness to and support of their students' academic needs.

Table 4.3

Indicators of the Deep Learning and Engagement Practices and Teacher Support Scales

			How often do these things happen in this mathematics class?
Deep Learning and Engagement Practices			
	SMQ M (SD)	SEOY M (SD)	**1 (never), 2 (rarely –a few times a year), 3 (sometimes – once or twice a month), 4 (often – once or twice a week), 5 (all or almost all lessons)**
RF1	4.2 (1.07)	4.0 (1.16)	*The teacher has us explain and justify our method for solving a problem.*
RF2	4.1 (1.05)	4.0 (1.07)	*The teacher has us consider multiple representations in solving a problem (for example: numbers, tables, graphs, pictures).*
RF3	3.1 (1.40)	3.3 (1.35)	*The teacher asks us to present our solution strategies to the rest of the class.*
RF4	3.4 (1.26)	3.5 (1.24)	*The teacher has us compare and contrast different methods for solving a problem.*
RW	3.5 (1.31)	3.6 (1.26)	*The teacher makes connections between the math and real-world situations or applications.*
GRP	3.2 (1.38)	3.3 (1.35)	*The teacher has us work in small groups to come up with joint solutions to a problem or task.*
			1 (never or almost never), 2 (some lessons), 3 (most lessons), 4 (almost every lesson)
CA1	2.8 (.87)	2.8 (.91)	*The teacher asks questions that make us reflect on the problem.*
CA2	2.9 (.85)	3.0 (.84)	*The teacher gives problems that require us to think for an extended time.*
CA3	2.4 (.94)	2.6 (.95)	*The teacher asks us to decide on our own procedures for solving complex problems.*
CA4	2.7 (.93)	2.9 (.88)	*The teacher gives problems that can be solved in several different ways.*
CA5	3.1 (.93)	3.1 (.92)	*The teacher helps us learn from the mistakes we have made.*
CA6	2.9 (.94)	2.9 (.92)	*The teacher presents problems in different contexts so that students know whether they have understood the concepts.*
CA7	3.0 (.96)	3.0 (.92)	*The teacher asks us to explain how we have solved a problem.*
CA8	3.0 (.89)	3.0 (.89)	*The teacher presents problems that require students to apply what they have learned to new contexts.*
CA9	2.6 (.90)	2.7 (.91)	*The teacher presents problems for which there is no immediately obvious method of solution.*
Teacher Support			
	SMQ M (SD)		**1 (never), 2 (rarely –a few times a year), 3 (sometimes – once or twice a month), 4 (often – once or twice a week), 5 (all or almost all lessons)**
TS1	3.8 (1.25)		*The teacher gives students an opportunity to express opinions.*
TS2	4.3 (1.06)		*The teacher gives extra help when students need it.*
TS3	4.0 (1.20)		*The teacher continues teaching until the students understand.*
TS4	4.0 (1.19)		*The teacher shows an interest in every student's learning.*
TS5	4.4 (1.04)		*The teacher helps students with their learning.*

Note. SMQ = Student Mathematics Questionnaire, SEOY = Student End of Year Questionnaire, RF = reform-oriented practices (source: 2012 NSSME scale, adapted), RW = real-world, GRP = group, CA = cognitive activation (source: 2012 PISA scale), TS = teacher support (source: 2012 PISA scale).

Mathematics course grade. This study used district-reported mathematics course grades from the fall 2017 semester which were converted to a grade points ranging from 4.3 (A+) to 0 (F).

Covariates. The same covariates used in Study 1 were utilized in this study: gender (1 = female), school (1 = Ace High School), advanced math (1 = advanced course, 0 = not advanced course), and off track in math (1 = off track, 0 = not off track).

Table 4.4

Bivariate Correlations

	School	Gender	Off Track in Math	Advanced Math	Math Comfort	Problem Solving	Math Interest	Teacher Support	DLEPs F	DLEPs S
Gender	-0.009									
Off Track in Math	0.046	-0.076**								
Advanced Math	-0.098***	0.074**	-0.285***							
S Math Comfort	-0.149***	-0.112***	-0.074**	0.224***						
S Problem Solving	-0.095***	-0.058*	-0.060*	0.178***	0.434***					
MF Math Interest	-0.040	-0.077**	-0.051	0.205***	0.469***	0.280***				
Teacher Support	-0.019	0.066*	-0.022	0.164***	0.153***	0.128***	0.361***			
DLEPs F	-0.085**	0.016	-0.045	0.258***	0.151***	0.194***	0.412***	0.634***		
DLEPs S	-0.103***	0.028	-0.057*	0.111***	0.176***	0.272***	0.267***	0.361***	0.494***	
Math Course Grade	-0.029	0.098***	-0.084**	0.260***	0.392***	0.219***	0.264***	0.188***	0.135***	0.103***

Note. DLEPs = Deep Learning and Engagement Practices. Reference category for school is Victory. Reference category for gender is male. F = fall; MF = mid-fall; S = spring.
$* p < .05$, $** p < .01$, $*** p < .001$.

Data Analytic Strategy

Structural equation modeling was utilized to investigate the relationships between select classroom experiences (teacher support and fall DLEPs) and students' mathematics identity beliefs (mathematics comfort and openness to problem solving). First, a measurement model tested the relationships between all latent variables and their corresponding indicators. After confirming the factor structure for mathematics comfort, openness to problem solving, mathematics interest, teacher support, and Deep Learning and Engagement Practices, structural pathways were added to the measurement model to test the hypothesized relationships among measures. The structural models tested (1) the direct paths from students' fall classroom experiences to their mathematics identity beliefs, and (2) the indirect paths from students' fall classroom experiences to their identity beliefs by way of three mediators: fall mathematics interest, fall mathematics course grade, and spring DLEPs (Figure 4.2). Also included were pathways from the covariates (school, gender, off track in math, and advanced math) to all latent variables and to fall mathematics course grade. However, these pathways were only retained in the models if observed to be statistically significant ($p < .05$).

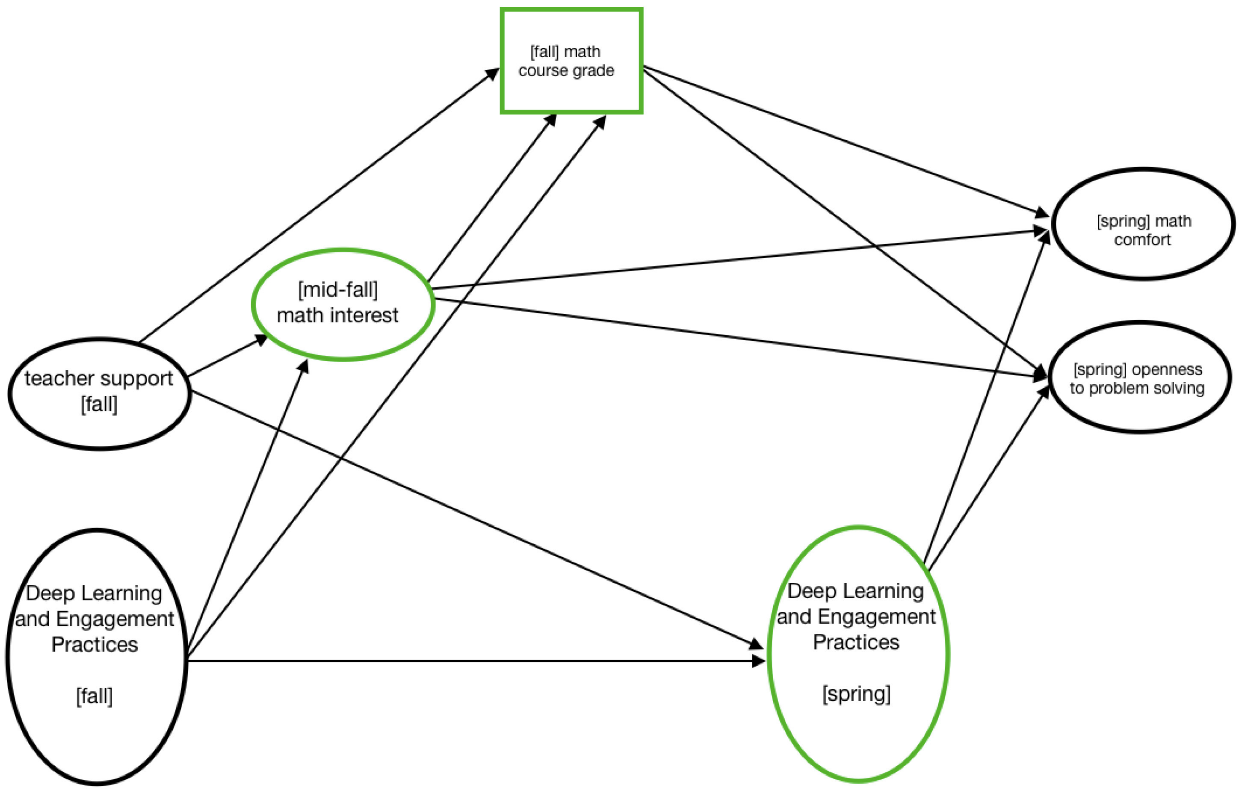

Figure 4.2. Indirect paths from students' fall classroom experiences to mathematics identity beliefs via mathematics interest, course grade, and spring Deep Learning and Engagement Practices. Color added to emphasize mediating variables.

Results

Measurement Model

To confirm that all manifest variables loaded onto their respective latent variable, a confirmatory factor analysis for the latent variables of mathematics comfort (SBQ and SEOY), openness to problem solving (SBQ and SEOY), mathematics interest (SBQ and SMQ), teacher support (SMQ), and DLEPs (SMQ and SEOY) was conducted. All indicators loaded onto their respective factors, with factor loadings ranging from .44 to .91. (Model fit: $x^2/df = 2.83$; CFI = .906; TLI = .900; RMSEA = .037; SRMR = .054.)

Structural Models

Table 4.5 presents the standardized coefficients and standard errors of all pathways included in the two structural models.

Total effects model. An initial model first investigated the direct effects of fall classroom experiences on students' spring mathematics comfort and openness to problem solving, after accounting for each identity measure at baseline, fall mathematics course grade, and all covariates (however only significant covariates remained in the total effects model). A positive association between fall DLEPs and students' openness to problem solving was observed ($\beta = .10$, $p < .05$), however the hypothesized association between fall DLEPs and students' mathematics comfort was not observed. Furthermore, no significant association between teacher support and either of the mathematics identity belief outcomes was observed. Fall mathematics course grade significantly predicted both mathematics comfort ($\beta = .21, p < .001$) and openness to problem solving ($\beta = .08, p < .01$). The total effects model explained 51.5% of the observed variance in students' mathematics comfort and 50.3% of the observed variance in students' openness to problem solving, with most fit indices (all but the TLI) indicating an adequate model fit to the data ($x^2/df = 3.32$; CFI = .901; TLI = .893; RMSEA = .042; SRMR = .061).

Table 4.5

Model Fit Indices and Standardized Parameter Estimates for Path Coefficients

	Total Effects Model β (*SE*)	**Mediation Model** β (*SE*)
Spring Mathematics Comfort		
S DLEPs	–	.06 (.04)
Fall DLEPs	.02 (.06)	-.07 (.07)
Teacher Support	.03 (.06)	-.02 (.06)
F Mathematics Course Grade	.21*** (.03)	.20*** (.03)
MF Mathematics Interest	–	.18*** (.04)
BL Mathematics Comfort	.61*** (.05)	.54*** (.06)
School	-.09*** (.07)	-.09*** (.07)
Spring Openness to Problem Solving		
S DLEPs	–	.20*** (.04)
Fall DLEPs	.10* (.06)	-.02 (.07)
Teacher Support	-.05 (.06)	-.05 (.06)
F Mathematics Course Grade	.08** (.03)	.07** (.03)
MF Mathematics Interest	–	.04 (.03)
BL Openness to Problem Solving	.67*** (.06)	.65*** (.06)
Gender	-.06* (.07)	-.05* (.07)
School	-.10*** (.07)	-.08** (.07)
Spring *DLEPs*		
Teacher Support	–	-.02 (.06)
Fall DLEPs	–	.56*** (.07)
School	–	-.07** (.06)
Fall *DLEPs*		
Advanced Math	.25*** (.06)	.25*** (.06)
School	-.06** (.04)	-.06** (.04)
Teacher Support		
Advanced Math	.15*** (.06)	.15*** (.06)
Fall Mathematics Course Grade		
F DLEPs	–	-.09 (.07)
MF Mathematics Interest	–	.13** (.03)
Teacher Support	–	.15** (.06)
BL Mathematics Comfort	.18*** (.04)	.12** (.05)
BL Openness to Problem Solving	.12*** (.04)	.10** (.04)
Advanced Math	.16*** (.08)	.16*** (.08)
Gender	.14*** (.07)	.14*** (.07)
Mid-fall Mathematics Interest		
F DLEPs	–	.33*** (.06)
Teacher Support	–	.06 (.06)
BL Mathematics Comfort	–	.14*** (.05)
BL Openness to Problem Solving	–	-.01 (.05)
BL Mathematics Interest	–	.55*** (.07)
Baseline Mathematics Comfort		
Advanced Math	.31*** (.07)	.32*** (.07)
Gender	-.18*** (.06)	-.18*** (.05)
School	-.06* (.05)	-.05* (.05)
Baseline Openness to Problem Solving		
Advanced Math	.23*** (.07)	.23*** (.07)
Baseline Mathematics Interest		
Advanced Math	–	.17*** (.07)
Gender	–	-.12*** (.06)

Note. Standardized coefficients (β) and robust standard errors (*SE*) are reported. BL = baseline; MF = mid-fall; S = spring; DLEPs = Deep Learning and Engagement Practices. Reference category for gender is male. Reference category for school is Victory. $n = 1139$.
$^{*}p < .05$, $^{**}p < .01$, $^{***}p < .001$.

Mediation model. The total effects model was augmented to include indirect pathways from fall classroom experiences to mathematics identity beliefs by way of students' mathematics interest, fall mathematics course grade, and spring DLEPs. Overall, model fit for the mediation model was acceptable ($x^2/df = 2.89$; CFI = .895; TLI = .889; RMSEA = .037; SRMR = .060), with the CFI and TLI slightly below the $.900$ cutoff for adequate model fit; all other fit indices were in the acceptable range. The mediation model explained 53.5% of the variance in mathematics comfort and 51.9% of the variance in openness to problem solving.

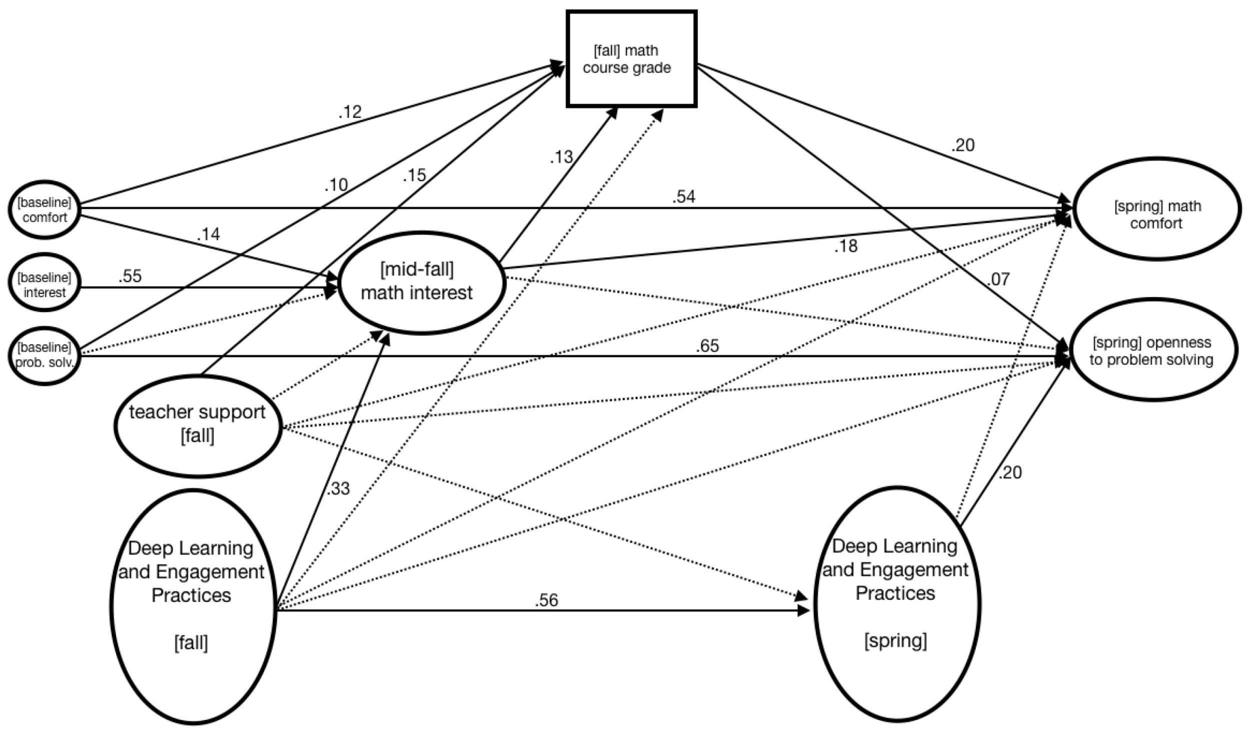

Figure 4.3. Significant path coefficients for mediation model. Observed indicators are represented by squares and latent variables by ovals. $n = 1139$. Standardized path coefficients are reported. A solid line indicates significant ($p < .05$) estimates. A dotted line indicates nonsignificant estimates. Pathways and estimates for model covariates omitted for clarity.

Direct effects.

Spring mathematics identity beliefs. The addition of the spring measure of DLEPs removed the significant association between fall DLEPs and openness to problem solving observed in the total effects model. Among covariates, after accounting for fall mathematics course grade, mathematics interest, teacher support, and fall and spring DLEPs, students attending Ace High School reported less mathematics comfort ($\beta = -.09, p < .001$) and less openness to problem solving ($\beta = -.08, p < .01$) compared to those attending Victory High School. Female students reported less openness to problem solving ($\beta = -.06, p < .05$) than male students. The other covariates (off track in math designation and advanced math designation) were not significantly associated with students' mathematics identity beliefs.

Classroom experiences. The students at Ace High School reported less frequent use of DLEPs by their mathematics teachers in both the fall ($\beta = -.06, p < .01$) and spring ($\beta = -.07$, $p < .01$) semesters. Students in an advanced mathematics course reported more frequent DLEPs in their classes in the fall semester ($\beta = .25, p < .001$) compared to those taking non-advanced mathematics courses; this difference was not observed in the spring semester. Additionally, students in advanced mathematics courses also reported significantly higher teacher support ($\beta =.15, p < .001$).

Mathematics course grade. Significantly higher grades were earned by students taking an advanced mathematics course ($\beta = .16, p < .001$) compared with those who were not, as well as by female students ($\beta = .14, p < .001$). After controlling for fall classroom experiences and mid-fall interest, significant positive associations were observed between mathematics course grade and two baseline self-beliefs: mathematics comfort ($\beta = .12, p < .01$) and openness to problem solving ($\beta = .10, p < .01$).

Mid-fall mathematics interest. Baseline mathematics comfort significantly predicted mid-fall mathematics interest ($\beta = .14, p < .001$), after accounting for fall classroom experiences, baseline mathematics interest, and baseline openness to problem solving.

Baseline belief measures. Compared to their peers in non-advanced mathematics courses, students in an advanced mathematics class reported greater baseline levels of mathematics comfort ($\beta = .32$, $p < .001$), openness to problem solving ($\beta = .23$, $p < .001$), and mathematics interest ($\beta = .17$, $p < .001$). Additionally, female students reported less baseline mathematics comfort ($\beta = -.18$, $p < .001$) and interest ($\beta = -.12$, $p < .001$) than male students.

Indirect effects. The mediation model estimated 14 indirect pathways between the two fall classroom experience variables (DLEPs and teacher support) and the two spring mathematics identity outcomes (mathematics comfort and openness to problem solving) via three mediators (mid-fall mathematics interest, fall mathematics course grade, and spring DLEPs). Each indirect effect was calculated by multiplying the path coefficient of the fall classroom experience variable to the mediator variable by the path coefficient of the mediator variable to the mathematics outcome variable. All indirect pathways are presented in Table 4.6.

Mathematics interest. The mediating role of mathematics interest on the effects between fall classroom experiences on three outcomes (mathematics comfort, openness to problem solving, and first semester mathematics course grade) was examined. As shown in Table 4.5 and Figure 4.3, when students experienced greater fall DLEPs, they reported greater mathematics interest ($\beta = .33$, $p < .001$). Mathematics interest was in turn positively associated with fall semester mathematics course grade ($\beta = .13$, $p < .01$) and spring mathematics comfort ($\beta = .18$, $p < .001$). Tests of indirect effects revealed that mid-fall mathematics interest mediated the associations between fall DLEPs and fall course grade ($\beta = .04$, $p < .01$) and between fall DLEPs and spring mathematics comfort ($\beta = .06$, $p < .001$).

Table 4.6

Indirect Effects via Mathematics Interest, Course Grade, and Spring DLEPs

	Mediation Model β (*SE*)
Mathematics Interest	
F DLEPs → MF Mathematics Interest → S Mathematics Comfort	.06*** (.02)
F DLEPs → MF Mathematics Interest → S Problem Solving	.01 (.02)
F DLEPs → MF Mathematics Interest → F Mathematics Course Grade	.04** (.02)
Teacher Support → MF Mathematics Interest → S Mathematics Comfort	.01 (.01)
Teacher Support → MF Mathematics Interest → S Problem Solving	.00 (.00)
Teacher Support → MF Mathematics Interest → F Mathematics Course Grade	.01 (.01)
Mathematics Course Grade	
F DLEPs → F Mathematics Course Grade → S Mathematics Comfort	-.02 (.02)
F DLEPs → F Mathematics Course Grade → S Problem Solving	-.01 (.01)
Teacher Support → F Mathematics Course Grade → S Mathematics Comfort	.03** (.01)
Teacher Support → F Mathematics Course Grade → S Problem Solving	.01* (.01)
Deep Learning and Engagement Practices	
F DLEPs → S DLEPs → S Mathematics Comfort	.03 (.03)
F DLEPs → S DLEPs → S Problem Solving	.11*** (.03)
Teacher Support → S DLEPs → S Mathematics Comfort	-.00 (.01)
Teacher Support → S DLEPs → S Problem Solving	-.00 (.02)

Note. Standardized coefficients (β) and robust standard errors (*SE*) are reported. BL = baseline; MF = mid-fall; S = spring; DLEPs = Deep Learning and Engagement Practices. n = 1139.
* $p < .05$, ** $p < .01$, *** $p < .001$.

Mathematics course grade. A significant positive relationship between teacher support and fall mathematics course grade was observed ($\beta = .15, p < .01$), and mathematics course grade was a significant predictor of both spring identity belief outcomes ($\beta = .20, p < .001$), mathematics comfort; $\beta = .07, p < .01$, openness to problem solving). Both indirect pathways from teacher support to spring mathematics comfort and openness to problem solving via mathematics course grade were significant (mathematics comfort, $\beta = .03, p < .01$, openness to problem solving, $\beta = .01, p < .05$).

Spring DLEPs. Unsurprisingly, a strong positive association between fall and spring DLEPs was observed ($\beta = .56, p < .001$). Spring DLEPs were in turn positively associated with spring openness to problem solving ($\beta = .20, p < .001$). This indirect pathway from fall DLEPs to openness to problem solving via spring DLEPs was significant ($\beta = .11, p < .001$).

Discussion

By exploring the relationships between students' mathematics classroom experiences and their mathematics identity beliefs, this investigation sought to identify potential supports for the mathematics identity aspect of students' productive disposition towards mathematics (i.e., "to view oneself as an effective learner and doer of mathematics," NRC, 2001). This study observed several direct and indirect effects among the classroom experiences and the two outcomes of mathematics comfort and openness to problem solving. Interestingly, neither teacher support nor Deep Learning and Engagement Practices directly predicted students' mathematics comfort. Rather, these mathematical experiences were indirectly associated with students' mathematics comfort through their associations with fall course grade (for teacher support) and mathematics interest (for DLEPs). That is, students who experienced greater teacher support in the fall went on to earn higher grades in mathematics that semester, which in turn predicted greater comfort with mathematics (and openness to problem solving), and students who experienced greater DLEPs in the fall demonstrated more interest in mathematics, which predicted significant increases to mathematics comfort (as well as better fall course performance).

One key aspect of student-teacher interactions in the classroom is the teacher's ability to cultivate a classroom climate that is emotionally supportive (Reyes et al., 2012). Teacher support has been found to be a strong predictor of achievement (LaRocque, 2008; Rimm-Kaufman & Chiu, 2007) and engagement (Birch & Ladd, 1997; Kaplan et al., 2011; Klem & Connell, 2004; Reyes et al., 2012; Stipek & Chiatovich, 2017). The link between the Deep Learning and Engagement Practices and students' openness to problem solving was more evident. When students experienced more frequent DLEPs in the fall, they were more likely to experience more frequent DLEPs in the spring, which strongly predicted students' end-of-year openness to problem solving. These findings are in line with previous research demonstrating the importance of student-centered, cognitively demanding mathematical tasks in engaging students and cultivating productive dispositions in mathematics (Dohn, 2013; Durik & Harackiewicz, 2007; Gaspard et al., 2015; Renninger & Bachrach, 2015; Stipek et al., 1998; Urdan & Schoenfelder, 2006; Watt, 2004).

After accounting for the classroom experiences variables, mathematics course grade, and other learning contextual factors (e.g., mathematics trajectory, advanced learning context), students attending Ace High School reported less spring openness to problem solving and less mathematics comfort compared with those at Victory High School. Furthermore, these differences in mathematics identity beliefs were greater in magnitude than those observed at the beginning of the year. Despite certain similarities between the two groups of students, school differences existed at the beginning of the year, and these findings suggest that those differences played a role in the widening disparity in mathematics comfort and openness to problem solving between the students at Ace High School and those at Victory High School. One such difference may be disparities in mathematics teacher quality; the impacts of instructional practices and other aspects of the learning environment (e.g., teacher support) is not necessarily equivalent for all students. For example, the mathematics teachers at Victory High School had more years of teaching experience (on average) than those at Ace High School. A study conducted by Aaronson et al. (2007), which investigated the effects of teacher quality on ninth grade mathematics performance, found that teacher quality had a much greater impact on test score gains for the students who entered the ninth grade with low and medium test scores than for the higher performing students. Similar results from an analysis of elementary students in Tennessee (Sanders & Rivers, 1996) found that as teacher effectiveness increased, the lowest-performing students were the first to benefit. A longitudinal study (Stipek & Chiatovich, 2017) investigating whether classroom climate and the quality of mathematics and reading instruction could predict engagement and academic performance of low-income students found that instructional quality positively predicted reading and mathematics achievement for students who previously had low academic performance. However, this effect was not observed among the high-performing students.

Students in an advanced mathematics course reported significantly greater mathematics comfort and openness to problem solving at the beginning of the year compared to those not taking an advanced mathematics course. However, these differences were not observed at the end of the year after accounting for teacher support, DLEPs (fall and spring), mathematics course grade, school, gender, mathematics interest, and baseline identity beliefs. Furthermore, the differences in mathematics interest observed at the beginning of the year between the two groups were not evidenced in mid-fall once the fall teacher support and DLEPs had been taken into account. These findings serve as further evidence of the relationship between students' classroom experiences and the motivational constructs that are fundamental to productive dispositions towards mathematics (mathematics interest, mathematics comfort, and openness to problem solving). It is important to note that the students enrolled in advanced mathematics courses reported significantly higher teacher support and more frequent DLEPs in their fall classes than did students enrolled in non-advanced courses, suggesting that differences in students' mathematics identity beliefs are influenced by the differential experiences taking place in advanced and non-advanced classes. This has important implications for schools looking to increase mathematics achievement and engagement, who may unknowingly be reproducing educational inequities by limiting the advanced course offerings available to their students. These findings provide insight into areas that could be improved within classrooms to better support the teaching and learning of mathematics, particularly with students traditionally excluded from advanced course offerings. Mathematics courses for secondary students vary in quantity and complexity of course content and in expectations regarding learning, pace, and ability. Even in upper-level mathematics courses, learning opportunities are inequitable (Dossey et al., 2016). We must consider how the classroom culture shapes how students are expected to work with one another, challenge each other's ideas, and make meaningful contributions, especially in mathematics classrooms where students are grouped by perceived ability ascribed to students by adults. Low socioeconomic status (SES) and ethnic minority students report less social support and less promotion of student collaboration and autonomy by teachers (Wang & Eccles, 2012), and student perceptions of the classroom environment tend to be less favorable among low SES, low-performing, and ethnic minority students (Battistich et al., 1995). Classrooms that prioritize students' memorization and recall of content are much more common in under-resourced schools with high proportions of minority students (Muller et al., 2010; Oakes et al., 2018), and students from marginalized groups are too often placed on mathematics tracks (course progression pathways) that limit their access to highly qualified mathematics teachers (Nasir, 2016) and fail to prepare them for continued study of fundamental mathematical concepts (NCTM, 2018). The practices that teachers of mathematics choose to employ in their classrooms are influenced in part by their own knowledge (e.g., of mathematics pedagogy; Hill et al., 2005) and by their belief systems, such as their beliefs about student learning capabilities (Borko & Putnam, 1996). Prior research has established links between differential treatment by race and subsequent academic outcomes and beliefs (Benner et al., 2015). Teachers' discriminatory behaviors toward students undermine the trust, belonging, and connectedness to school and schooling that nurture student well-being and positive academic outcomes (Benner et al., 2015; Eccles & Roeser, 2011). More research is needed to examine the ideological underpinnings of mathematics curricula for advanced and non-advanced courses and assess whether and to what extent those differences influence the ways in which students are encouraged to engage with mathematics.

Consistent with prior research, gender differences were observed in the mathematics motivational variables. Boys frequently report more mathematics interest, greater perceived importance of mathematics in their lives, and higher perceived mathematics ability in comparison with girls (Crombie et al., 2005; Meece et al., 1990; Watt, 2004). However, although female students reported significantly less interest in mathematics and less comfort with mathematics than male students at the beginning of the year, no significant difference in mid-fall mathematics interest (after accounting for fall DLEPs, teacher support, and all baseline motivational constructs) or spring mathematics comfort (once teacher support, DLEPs, mathematics course grade, school, mid-fall mathematics interest, and baseline comfort had been taken into account) was observed. This implies that, after controlling for aspects of classroom context like teacher support and DLEPs, in addition to demographic characteristics, students tend to feel equally comfortable with and interested in mathematics irrespective of their gender. However, females did report a somewhat lower openness to problem solving in the spring despite no observed gender differences in openness to problem solving at baseline. Specifically, female students reported less openness to problem solving at the end of the year compared to males, after taking account of all the paths in the model including female students' lower baseline mathematics comfort, lower baseline mathematics interest, and higher fall mathematics course grades. It is unclear why this reduction in their problem-solving confidence (not limited to mathematics problems) emerged at the same time their comfort with and interest in mathematics was increasing. It could be that female students' distinctive experiences in other courses and subject areas, gender-specific life problems, or constraints they encountered throughout the year weakened their overall identity as problem solvers.

Conclusion

When students rarely encounter stimulating mathematical problems, they come to view success in mathematics as a product of memorization of facts and procedures rather than sense making (Boaler, 2015a; Boaler & Greeno, 2000; Schoenfeld, 1989; 2009), a perspective that can cause them to lose confidence in themselves as learners (NRC, 2001). Student-centered teaching practices are vital to the active engagement of students (Zepke, 2011). The ways in which teachers ask questions can support or limit informational and interpersonal aspects of participation (Gresalfi et al., 2012). Students positioned as skilled and capable sense-makers can reframe or disrupt how they identify with mathematics. When students know they are expected to support and challenge one another, their responsibility for meaning-making increases (Gresalfi, 2009). Understandings develop as students build on prior understandings, challenge misconceptions, and reflect with purpose on what they are learning (Letwinsky & Cavender, 2018; Perkins, 1998). In a classroom where students are expected to attend to the learning of one another and to engage collaboratively with the content, what it means to be a competent member of the classroom is redefined (Gresalfi et al., 2008). Teachers who respect their students and provide a learning environment where students are encouraged to express their opinions foster the type of socio-emotional support students need to engage with and persist on academic learning tasks (Wang, 2012; Wigfield et al., 2006).

The tendency of much motivational research to focus on the individual as the site of remediation (e.g., an intervention to help students to understand the importance of perseverance in learning mathematics) is problematic, insofar as it minimizes the extent to which students' mathematics identity beliefs stem from their school and classroom experiences and can be advanced or eroded by particular classroom practices (Gresalfi, 2009). This is the first study that jointly considers differing aspects of the student learning experience – reform-oriented and cognitively demanding teacher practices, collaborative learning opportunities, and application of mathematics content to real-world contexts – as a single, holistic, multidimensional measure of deeper learning and engagement in the mathematics classroom. After accounting for other contextual variables (such as teacher support and prior mathematics identity beliefs), students who experienced more DLEPs reported greater interest in mathematics, which in turn was associated with increases in both mathematics course performance and mathematics comfort. Furthermore, the DLEPs factor strongly predicted students' openness to problem solving. The unique contribution of DLEPs to differences in mathematics identity beliefs and the mechanisms by which DLEPs advance those beliefs, as observed and reported in this study, offer a valuable starting point for future research on productive disposition, an essential component of mathematics proficiency.

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CHAPTER V

This dissertation's conceptual framework (Figure 5.1) positions the mathematics classroom context as a primary influencer of how students come to identify or disidentify with mathematics. Two investigations examined how aspects of students' dispositions towards mathematics (mathematics valuation beliefs, Study 1; mathematics identity beliefs, Study 2), were impacted by select pedagogical practices and mathematical experiences occurring in their classrooms.

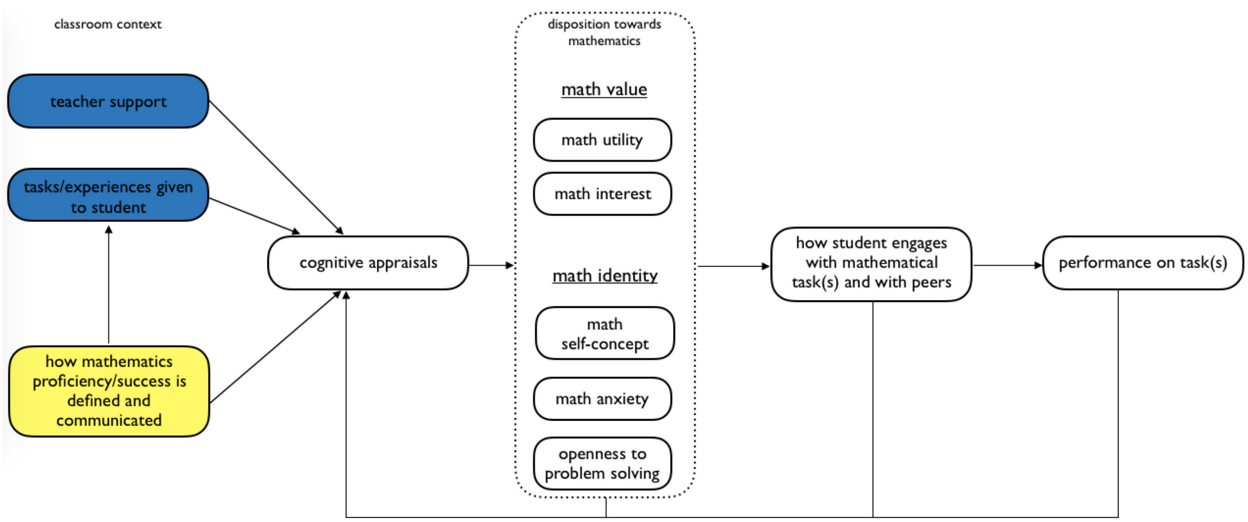

Figure 5.1. The Mathematics Disposition Framework. Color added to indicate the elements of the mathematics *classroom context* that are considered to influence students' dispositions towards mathematics: blue represents the constructs or variables investigated in this dissertation's structural models, yellow represents unmeasured constructs, processes, or behaviors hypothesized to occur.

It is critical to consider dispositions as dynamic; students' productive dispositions towards mathematics are not *determined* by learning contexts, but rather shift and change through their engagement with particular teaching practices and mathematical experiences. The two studies presented in Chapters 3 and 4 documented how elements of the mathematical learning context directly advanced productive disposition or indirectly supported its development. Research has yet to explain why classroom practices impact students differently or which aspects of instructional practices advance the development of different learning dispositions among learners in the same classroom (Gresalfi, 2009) or how students' dispositions towards mathematics shift within and across activities. Furthermore, complex interrelationships between productive disposition constructs exist. While prior research asserts that students' competence beliefs account for changes in mathematics values (e.g., Jacobs et al., 2002), it is possible that relationships between competence beliefs and mathematics values operate in the opposite direction (Watts, 2004), as was observed in Study 2, with Deep Learning and Engagement Practices advancing competence beliefs via mathematics interest. It is therefore important for future research to clarify the causal sequencing to better understand and explain students' developmental trajectories for these constructs. Nevertheless, the process model presented in this dissertation and the subsequent two investigations have sought to demonstrate the mechanisms by which changes to productive disposition can be understood, and how certain practices and mathematical experiences provide a space for the negotiation of students' mathematics identities.

The findings from these two investigations are not a sufficient basis for assembling a generalizable list of pedagogical practices that, if incorporated into classrooms, will advance all students' productive dispositions towards mathematics. A classroom rich with opportunities for students to engage deeply with mathematics does not mean that all students will choose to do so. While classroom practices can influence students' dispositions, students' histories engender their initial dispositions (Gresalfi, 2009). Additionally, because elements of classroom systems are interrelated (Staples, 2008), changing one aspect of a classroom system may do little to advance productive dispositions towards mathematics if other, oppositional aspects remain in place. If, for example, the grading system in a classroom rewards correctness over process or effort, teacher attempts to praise students for their perseverance or reasoning are unlikely to make a lasting impression on students. However, classroom structures that afford consistent, repeated opportunities for learners to engage meaningfully with the mathematical context are far more likely to advance productive disposition than those that do not.

While this work did not explicitly measure the role of achievement goals on students' dispositions towards mathematics, achievement goal theory informed this dissertation's conceptual framework. It is therefore of value to consider how these findings connect to research on goal orientations. According to Dweck and colleagues (e.g., Dweck, 1986; Dweck & Leggett, 1988), students' implicit theories of intelligence serve as the cognitive frameworks that position them to pursue certain goals in achievement situations. Goal orientations account for the ways in which students make sense of events such as failure and react to those events (e.g., formulating a plan of action in response, withdrawing). Individuals who believe that mathematics ability is a trait that cannot be substantially enhanced (entity theorists) choose different achievement goals than incremental theorists, who believe that everyone can achieve substantial growth with the right motivation, opportunity, and instruction. There is substantial evidence that students' implicit theories predict their goals in achievement settings (De Castella & Byrne, 2015; Dinger et al., 2013; Smiley et al., 2016), and that goals predict ability attributions (Ames & Archer, 1988; Grant & Dweck, 2003; Smiley et al., 2016).

Personal goals are separated into performance and mastery goal orientations (Elliott & Dweck, 1988), with avoidance and approach facets (Elliot & McGregor, 2001), and play a role in learning situations, learning outcomes, and students' behavior (Pintrich, 2003). Students' maladaptive responses to failure or academic setbacks (e.g., experiencing negative emotions, avoiding challenge, blaming low ability, loss of interest) would suggest that they are guided by a performance goal (i.e., are focused on validating or demonstrating their competence) in that particular achievement context. In contrast, students who demonstrate adaptive behaviors in response to setbacks (e.g., remain optimistic, monitor progress, employ a variety of problem-solving strategies) are more likely focused on increasing competence (i.e., they hold a mastery goal). Individuals with mastery goals apply renewed effort after failure because they believe that effort is constructive, whereas individuals with ability goals withdraw effort because they believe effort is inefficacious. Associations have been observed between performance goals and loss of interest/enjoyment and between mastery goals and continued interest/enjoyment (Dweck & Leggett, 1988; Grant & Dweck, 2003; Smiley et al., 2016). Mastery goals have also been found to correlate with utility and interest value (Daniels et al., 2009; Dweck & Leggett, 1988; Grant & Dweck, 2003; Hulleman et al., 2008; MacIver et al., 1991; Senko & Harackiewicz, 2005; Smiley et al., 2016; Wigfield & Eccles, 2002). Mathematics classrooms' goal structures play a role in the development of students' beliefs, particularly regarding the nature of mathematics and mathematics learning. Continued research is needed to examine classroom goal structures alongside the elements of the classroom context investigated here, in order to discern more fully how the ways classrooms are organized come to influence students' dispositions towards mathematics.

Limitations

The studies presented here used measures of mathematics interest, mathematics utility, mathematics comfort, and openness to problem-solving to investigate two sides of productive disposition, as defined by the NRC (2001): "to perceive [mathematics] to be worthwhile and useful" (measured by interest and utility, Study 1), and "to view oneself as an effective doer and learner of mathematics" (measured by comfort and openness to problem solving, Study 2). Consequently, the empirical results of these analyses are conditional in that they were based on these particular operationalizations. Additionally, all mathematics value and identity beliefs were measured using Likert scales. Likert scales inadequately capture nuances in respondents' thinking (Phillipp & Siegfried, 2015), and are especially problematic when items are presented in statement form and respondents are asked to rate their level of agreement or disagreement with the statement in question (the agree/disagree item format is considered susceptible to the introduction of additional error into participants' ratings; Dillman et al., 2014; Gehlbach & Artino, 2018). There is a need for continued research that incorporates qualitative forms of data collection in its design (i.e., open-ended survey questions, interviews, and classroom observations), with a particular focus on how productive disposition can best be measured and how it relates to assessments of students' mathematical competence (Gilbert, 2014; Schoenfeld, 2007).

The measures of each teacher practice/mathematical experience investigated in these studies relied on student reports (i.e., the extent to which the student perceived their teacher to be implementing a practice). Although students are arguably the ones best situated to describe their own teachers' typical instructional practices (teachers may overreport their use of research-based instructional strategies; Durham et al., 2018), and have been found to be capable of discriminating between the quality of different elements of the classroom environment (Nelson et al., 2017), students in the same learning environment tend to perceive teacher practices differently (Ames, 1992; Desimone et al., 2010; Patrick et al., 2011). This is supported by empirical findings that indicated only small-to-moderate levels of shared perceptions of classroom goal structures reported by students in the same class (Deemer, 2004; Miller & Murdock, 2007). Student reports will thus imperfectly reflect the actual frequency of each item's occurrence. Although the modeling methods used in this study took account of random measurement error in students' observations, they did not take account of unknown systematic errors that might have biased students' reports. Thus, the two studies' findings reflect the relationships between students' *individual perceptions* of the classroom context on their valuation of mathematics (i.e., interest and utility) and identity beliefs (i.e., mathematics comfort and openness to problem solving). Furthermore, these analyses were based on one year of students' mathematics classroom experience. Dispositions are fluid, changing over the course of the year.

The findings do not provide evidence that observed advancements in students' productive dispositions lasted over longer periods of time.

The paths in the structural models represent hypothesized causal effects of teacher practices and mathematical experiences on students' disposition outcomes. While the structural models are specifying and estimating the magnitude and direction of causal impacts, the research design of the *Engaging High School Students in Academic Work* professional development study (Mac Iver et al., 2020) and the data available for these analyses cannot support causal inferences. As such, although the data have enabled modeling of the relationships among these variables over the course of a school year, definitive causal conclusions about these relationships are impossible. Nevertheless, these analyses were based on motivation research that supports such causality. These models should be replicated in the future using trained observers of teachers' practices and randomized designs where the elements of the classroom context of interest to these studies are systematically controlled as independent variables. Additionally, because student assignment to teacher and to mathematics course was not done at random, but determined by factors including grade, ascribed ability, and scheduling conflicts, and teachers' course assignments were also not made at random but determined at least in part by factors like seniority, credential type, and enrollment needs, it is likely there existed relationships between teacher practices and class composition unaccounted for in this study.

Findings are not generalizable to student populations that differ from this sample (see Chapter 2 for student demographics). A large percentage of students (56.6%) was eliminated from the statistical modeling because students did not complete all three student questionnaires, reducing the representativeness of the sample even further. The students who remained in the analytic sample differed from those who did not in ways that resulted in the under- or over-representation of certain student populations in the statistical models. The analytic samples in the two studies contained, among other differences, smaller proportions of students with disabilities, twelfth grade students, male students, and students classified as being behind in mathematics. Furthermore, the data loss mechanism for some variables was likely not missing at random (NMAR). Despite the inclusion of variables that explained much of the data loss (e.g., mathematics course grade, off track in math designation, advanced math designation, gender) and the employment of full maximum information likelihood estimation, it is improbable the path analyses contained in this dissertation yielded completely unbiased results.

Shifting Teacher Practice to Support Productive Disposition

Teaching mathematics effectively is a complex task that requires both procedural and conceptual support. The ability to weave together content and pedagogy in order to maximize understanding requires deep understanding of both mathematics and teaching (NCTM, 2014). Teacher beliefs have a significant impact on teacher practice (Pajares, 1992; Stipek et al., 2001; Wilkins, 2008). Future teaching intentions are shaped by beliefs pertaining to mathematics pedagogy and by teachers' experiences with mathematics teaching and learning (Letwinsky & Cavender, 2018). Efforts to shift teacher practices must therefore give careful consideration to teachers' past experiences and their beliefs and ideologies about teaching and learning.

It has been suggested that self-efficacy in teaching mathematics is a powerful influence on a teacher's receptivity to changing instructional practices, and that low self-efficacy can lead to avoidance of instructional practices and content (Letwinsky & Cavender, 2018; Tschannen-Moran & McMaster, 2009). Gregoire (2003) theorizes that the extent to which teachers are motivated to attempt and sustain new instructional practices depends whether they perceive professional development as a threat to their efficacy or as a challenge that will strengthen their efficacy. Interpersonal factors such as collaboration and support for professional growth from administrators and peers, the national policy context, and the influences of the school setting also play a role in the success of teacher professional development (Turner et al., 2011; Hochberg & Desimone, 2010).

Letwinsky and Cavender (2018) attribute teacher resistance to adopting the pedagogical methods required to support conceptual learning to a disconnect between how teachers themselves learned mathematics and the constructivist teaching strategies that promote conceptual understandings of mathematics content. Stipek and colleagues (2001) found that teachers who considered mathematics ability to be malleable also believed that mathematics instruction should equip students to use mathematics as a tool for thought and that autonomy should be given to students. In contrast, teachers that viewed mathematics ability as fixed valued mathematics teaching practices that prioritized correctness and procedures as well as strong control of student learning. An in-depth study of teachers' instructional practices conducted by Jacobson and Izsák (2015) found that orientations to learning the professional development content were influenced by teachers' valuation of the content and their self-concepts of ability. Although teacher motivation and knowledge accounted for the relationships between learning opportunities and instructional practice, once motivational effects were removed, the relationship between knowledge and practice was no longer present. To support teachers who place low value on the target learning and who have low self-concepts of ability, Jacobson and Izsák recommended that their professional learning experiences require a focus beyond the acquisition of knowledge. The findings reported in this paper suggest that similar considerations should guide the teaching of mathematics to students; learning cannot focus solely on the acquisition of content and procedural fluency, but must also address deficits in motivation and learning orientations.

Conclusion

This dissertation has attempted to frame a discussion around how productive disposition, a strand of mathematics proficiency, develops in relation to the classroom context. More specifically, this work connected students' mathematical experiences to two aspects of their mathematical dispositions: their *ideas* about mathematics and the ways in which they *see themselves* as capable doers and learners of mathematics. Both the Mathematics Disposition Framework and the findings from the two investigations offer a constructive foundation for future empirical research on productive disposition.

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Curriculum Vitae

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2011 M.A. Education, Claremont Graduate University
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PUBLICATIONS

2020 Grant, A., Hann, T. Godwin, R., Shackelford, D., & Ames, R. A Framework for Graduated Teacher Autonomy: Linking Teacher Proficiency with Autonomy. *Educational Forum*.
2019 Hann, T. Investigating the Impact of Teacher Practices and Non-Cognitive Factors on Mathematics Achievement. *Research in Education*. 108(1).
2019 Mac Iver, D., & Hann, T. Effective School Reforms for Increasing Engagement. In J. A. Fredricks, A. L. Reschly, & S. L. Christenson (Eds.), *Handbook of Student Engagement Interventions* (pp. 245–262). Academic Press.
2018 Godwin, R., Hann, T., & Sandmel, K. Examining School Elements through Collective Awareness: A Case Study of Six Students at a Special Education Day School. *International Journal of Technology and Inclusive Education*. 7(1).

PRESENTATIONS

2020 Paper Session *An Investigation of the Relationships Between Mathematics Classroom Practices and Self-motivational Beliefs.* American Educational Research Association (AERA). CA. (Cancelled).
2017 Paper session *Examining School Elements Through Collective Mention: A Case Study of Six Students at a Special Education Day School*. Canada International Conference on Education (CICE). ONT.
2017 Paper Session *Mathematics Engagement: Effective Practices for Urban Minority Students*. American Educational Research Association (AERA). TX.
2016 Poster Presentation *Graduated Autonomy for Teachers*. Eastern Evaluation Research Society (EERS). NJ.

TEACHING EXPERIENCE

2016 Number and Operations for K-8 Lead Teachers. Johns Hopkins University.

RESEARCH EXPERIENCE

2019 Equipping High School Teachers to Increase Student Motivation and Course Passing Rates
2017 Teacher Negotiation of Mathematics Curriculum and Pedagogy
2016 Case Studies: The School Experience of Elementary, Middle School, and High School Students with Language-Based Disabilities

PROFESSIONAL SERVICE

2018 Professional Development Series (mathematics). Ace High School (pseudonym), California.
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REFERENCES

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